A Hypothetical Mechanism of Generating Magnetically Charged Fermions by CP-symmetry Breaking

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Z. Naturforsch. **60a,** 696 – 718 (2005); received June 30, 2005

Based on the assumption that electroweak bosons, leptons and quarks possess a substructure of elementary fermionic constituents, in a previous paper it was demonstrated that under CP-symmetry breaking "electric" and "magnetic" electroweak bosons coexist, where the latter transmit magnetic monopole interactions. In this paper the calculation is extended to the derivation of the effective theory for electroweak bosons and leptons. It is shown that under the influence of CP-symmetry breaking charged leptons are transmuted into dyons, the interactions of which are mediated by electric and magnetic electroweak bosons. The dynamical law of the fermionic constituents is assumed to be given by a relativistically invariant nonlinear spinor field theory with local interaction, canonical quantization, selfregularization and probability interpretation. The corresponding effective theory is derived by means of weak mapping theorems and turns out to be an extension of the Standard model for dyons where owing to CP-violation SU(2)-symmetry is simultaneously broken. A mechanism of inducing CP-symmetry violation in the low energy range is proposed.

Key words: CP-symmetry Breaking; Magnetic Monopoles; Dyons; Effective Electroweak Theory.

1. Introduction

The existence of magnetic monopoles and dyons (electrically and magnetically charged particles) is compatible with present quantum field theoretic models in elementary particle physics. In case such monopoles and dyons exist, they would exert considerable influence on various elementary particle reactions, and in last decades an increasing number of theoretical papers was devoted to this topic.

This development was mainly promoted by the discoveries of t'Hooft [1] and Polyakov [2]. They demonstrated that in grand unified theories (GUTs) magnetic monopole states can be derived.

Such states have very heavy masses being out of reach for their production by accelerators. But in the early universe they could have catalyzed proton decay. The monopole induced baryon number violating processes [3, 4], or *other* monopole induced actions would be of great physical interest if they could be realized independently of the GUT scheme, [5]. So the first question is: Are there monopole models and reactions apart from this scheme?

As at present no one has any concrete experimental experience about the existence and the physical properties of magnetic monopoles and dyons, one is in no way obliged to follow the topological construction of monopole states by Dirac or t'Hooft and Polyakov.

Indeed, two decades ago Lochak proposed a massless neutrino [6], acting as a magnetic monopole and he described its electromagnetic action by means of a magnetic vector potential introduced by Cabibbo and Ferrari [7]. In this approach any topological property is avoided. In addition Lochak demonstrated that in de Broglie's photon theory magnetic photons can be derived which are to be associated with the magnetic vector potential [8]. But in de Broglie's photon theory either magnetic *or* electric photon states can be calculated which cannot exist simultaneously [9]. Furthermore the neutrino like magnetic monopoles are singular phenomena, the embedding of them in the modern elementary particle theory is unknown.

Therefore the following problems have to be treated:

(i) does a medium exist which transmits electric as well as magnetic monopole actions;

(ii) can one discover "elementary" or other particles which act as magnetic monopoles or dyons, respectively;

(iii) can the hypothetical medium and the hypothetical monopoles and dyons be incorporated into an extended electroweak Standard model?

To solve these problems we use a model which is based on a relativistically invariant nonlinear spinor field theory with local interaction, canonical quantization, selfregularization and probability interpretation. This model implies that in accordance with de Broglie and Heisenberg the present "elementary" particles are assumed to possess a fermionic substructure. This assumption is essential for our further proceeding as it offers a sufficient flexibility in finding new types of particles in accordance with (i) and (ii), but at the same time it excludes arbitrariness in treating (iii). The model is expounded in detail in [9] and special reference will be made to preceding papers in the following.

In [10] it was demonstrated that in this model electric and magnetic photon states can coexist if the CP-symmetry of the vacuum is violated, a result which is in accordance with phenomenological findings about CP-symmetry breaking and the existence of magnetic monopoles [7]. Thus this result can be considered as the step (i) and it can be used as a starting point for the subsequent investigations.

The symmetry breaking by the vacuum state is common to solid state physics and can be interpreted by considering the material groundstate of the system [11]. In high energy physics the symmetry breaking by the ground states is even a constituting element of the whole theory. But in constrast to solid state physics the physical cause for various symmetry breakings is poorly or not at all understood.

In analogy to solid state physics, we propose to connect CP-symmetry breaking in elementary particle physics with real physical groundstate properties. For instance in quantum electrodynamics one can directly observe the action and the modification of the physical vacuum in finite volumes by the Casimir effect [12]. A similar effect of the modification of the vacuum may occur if in finite volumes an electric discharge is set off leading to a plasma state which acts as a physical surrounding of single or several plasma particles and their reactions. This surrounding breaks CP-symmetry and if in this case reactions are considered the surrounding can be idealized as a CP-symmetry breaking background, i. e., vacuum.

The complete vacuum state is then given by the superposition of the original CP-symmetric vacuum state and its local CP-symmetry breaking part. For the short time of the discharge, due to the latter part of the vacuum, magnetic bosons are generated by the motion of the fermions and simultaneously couple to these fermions, transmuting them into magnetic monopoles or dyons, respectively.

In the following we do not bother about the time interval of the discharge, but simply consider the field dynamics if CP-symmetry is broken. Further comments are given in the summary, Section 8, referring to experiments which have been carried out by Urutskoev et al. [5].

2. Algebraic Representation of the Spinor Field

The algebraic representation is the basic formulation of the spinor field model. In order to avoid lengthy deductions we give only some basic formulas of this formalism and refer for details to [9, 13, 14]. The corresponding Lagrangian density reads (see [9], equation (2.52)).

$$\begin{aligned}\mathcal{L}(x) := \sum_{i=1}^{3} \lambda_i^{-1} \bar\psi_{A\alpha i}(x)(\mathrm{i}\gamma^\mu\partial_\mu - m_i)_{\alpha\beta}\delta_{AB}\psi_{B\beta i}(x) \\ - \frac{1}{2}g\sum_{h=1}^{2}\delta_{AB}\delta_{CD}v^h_{\alpha\beta}v^h_{\gamma\delta}\sum_{i,j,k,l=1}^{3}\bar\psi_{A\alpha i}(x)\psi_{B\beta j}(x)\bar\psi_{C\gamma k}(x)\psi_{D\delta l}(x)\end{aligned} \tag{1}$$

with $v^1 := \mathbf{1}$ and $v^2 := \mathrm{i}\gamma^5$. The field operators are assumed to be Dirac spinors with index $\alpha = 1,2,3,4$ and additional isospin with index $A = 1,2$ as well as auxiliary fields with index $i = 1,2,3$ for nonperturbative regularization. The algebra of the field operators is defined by the anticommutators

$$[\psi^+_{A\alpha i}(\mathbf{r},t)\psi_{B\beta j}(\mathbf{r}',t)]_+ = \lambda_i\delta_{ij}\delta_{AB}\delta_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}') \tag{2}$$

resulting from canonical quantization. All other anticommutators vanish.

To obtain a uniform transformation property with respect to Lorentz transformations, the adjoint spinors are replaced by formally charge conjugated spinors which are defined by

$$\psi^c_{A\alpha i}(x) = C_{\alpha\beta}\bar\psi_{A\beta i}(x) \tag{3}$$

and the index Λ is introduced by

$$\psi_{A\Lambda\alpha i}(x) = \begin{pmatrix} \psi_{A\alpha i}(x); \Lambda = 1 \\ \psi^c_{A\alpha i}(x); \Lambda = 2 \end{pmatrix} \tag{4}$$

Then the set of indices is defined by $Z := (A, \Lambda, \alpha, i)$.

In order to derive definite results from the theory a state space is needed in which the dynamical equations can be formulated. This is achieved by the use of the algebraic Schroedinger representation. For a detailed discussion we refer to [9, 13, 14].

To ensure transparency of the formalism we use the symbolic notation

$$(\psi_{I_1} \dots \psi_{I_n}) := \psi_{Z_1}(\mathbf{r}_1, t) \dots \psi_{Z_n}(\mathbf{r}_n, t) \tag{5}$$

with $I_k := (Z_k, \mathbf{r}_k, t)$. Then in the algebraic Schroedinger representation a state $|a\rangle$ is characterized by the set of matrix elements

$$\tau_n(a) := \langle 0|\mathcal{A}(\psi_{I_1} \dots \psi_{I_n})|a\rangle, \quad n = 1 \dots \infty, \tag{6}$$

where $\mathcal{A}$ means antisymmetrization in $I_1 \dots I_n$.

By means of this definition the calculation of an eigenstate $|a\rangle$ is transferred to the calculation of the set of matrix elements (6) which characterize this state. For a compact formulation of this method, generating functionals are introduced and the set (6) is replaced by the functional state

$$|\mathcal{A}(j;a)\rangle := \sum_{n=1}^{\infty} \frac{\mathrm{i}^n}{n!} \sum_{I_1 \dots I_n} \tau_n(I_1 \dots I_n|a) j_{I_1} \dots j_{I_n}|0\rangle_f, \tag{7}$$

where $j_I := j_Z(\mathbf{r})$ are the generators of a CAR-algebra with corresponding duals $\partial_I := \partial_Z(\mathbf{r})$ which satisfy the anticommutation relations

$$[j_I, \partial_{I'}] = \delta_{ZZ'}\delta(\mathbf{r} - \mathbf{r}'), \tag{8}$$

while all other anticommutators vanish.

With $\partial_I|0\rangle_f = 0$ the basis vectors for the generating functional states can be defined. The latter are not allowed to be confused with creation and annihilation operators of particles in physical state spaces, because generating functionals are formal tools for a compact algebraic representation of the states $|a\rangle$: To each state $|a\rangle$ in the physical state space we associate a functional state $|\mathcal{A}(j;a)\rangle$ in the corresponding functional space. The map is biunique and the symmetries of the original theory are conserved. For details see [9, 14].

In order to find a dynamical equation for the functional states, we apply to the operator products (5) the Heisenberg formula

$$\mathrm{i}\frac{\partial}{\partial t}\mathcal{A}(\psi_{I_1} \dots \psi_{I_n}) = [\mathcal{A}(\psi_{I_1} \dots \psi_{I_n}), H]_-, \quad n = 1 \dots \infty, \tag{9}$$

where H is the Hamiltonian of the system under consideration.

If $|0\rangle$ as well as $|a\rangle$ are assumed to be eigenstates of H, then between both states the matrix elements of (9) can be formed and subsequent evaluation of these expressions leads to the functional equation

$$E_0^a|\mathcal{A}(j;a)\rangle = [K_{I_1 I_2} j_{I_1} \partial_{I_2} - W_{I_1 I_2 I_3 I_4} j_{I_1} (\partial_{I_4}\partial_{I_3}\partial_{I_2} + A_{I_4 J_1} A_{I_3 J_2} j_{J_1} j_{J_2} \partial_{I_2})]|\mathcal{A}(j;a)\rangle \tag{10}$$

with $E_0^a = E_a - E_0$. For details of the derivation, see [9, 14].

The symbols which are used in (10) are defined by the following relations

$$K_{I_1 I_2} := \mathrm{i}D^0_{I_1 I}(D^k \partial_k - m)_{I I_2} \tag{11}$$

with

$$D^\mu_{I_1 I_2} := \mathrm{i}\gamma^\mu_{\alpha_1\alpha_2}\delta_{A_1 A_2}\delta_{\Lambda_1\Lambda_2}\delta_{i_1 i_2}\delta(\mathbf{r}_1 - \mathbf{r}_2), \quad m_{I_1 I_2} := m_{i_1}\delta_{\alpha_1\alpha_2}\delta_{A_1 A_2}\delta_{\Lambda_1\Lambda_2}\delta_{i_1 i_2}\delta(\mathbf{r}_1 - \mathbf{r}_2), \tag{12}$$

and

$$W_{I_1 I_2 I_3 I_4} := -\mathrm{i}D^0_{I_1 I} V_{I I_2 I_3 I_4} \tag{13}$$

with

$$V_{I_1I_2I_3I_4} := \sum_{h=1}^{2} g\lambda_{i_1} B_{i_2i_3i_4} v^h_{\alpha_1\alpha_2}\delta_{A_1A_2}\delta_{\Lambda_1\Lambda_2}(v^hC)_{\alpha_3\alpha_4}\delta_{A_3A_4}\delta_{\Lambda_31}\delta_{\Lambda_42}\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_1-\mathbf{r}_4),$$
$$B_{i_2i_3i_4} := 1, \quad i_2,i_3,i_4 = 1,2,3, \tag{14}$$

and the anticommutator matrices

$$A_{I_1I_2} := \lambda_{i_1}(C\gamma^0)_{\alpha_1\alpha_2}\delta_{A_1A_2}\sigma^1_{\Lambda_1\Lambda_2}\delta_{i_1i_2}\delta(\mathbf{r}_1-\mathbf{r}_2). \tag{15}$$

So far this functional equation holds for any algebraic representation. A special representation can be selected by specifying the corresponding vacuum. This can be achieved by the introduction of normal ordered functionals which are defined by

$$|\mathcal{F}(j;a)\rangle := \exp\left[\frac{1}{2}j_{I_1}F_{I_1I_2}j_{I_2}\right]|\mathcal{A}(j;a)\rangle =: \sum_{n=1}^{\infty}\frac{\mathrm{i}^n}{n!}\varphi_n(I_1\ldots I_n|a)j_{i_1}\ldots j_{I_n}|0\rangle_f, \tag{16}$$

where the two-point function

$$F_{I_1I_2} := \langle 0|\mathcal{A}\{\psi_{Z_1}(\mathbf{r}_1,t)\psi_{Z_2}(\mathbf{r}_2,t)\}|0\rangle \tag{17}$$

contains an information about the groundstate and thus fixes the representation.

The normal ordered functional equation then reads

$$E_0^a|\mathcal{F}(j;a)\rangle = \mathcal{H}_F(j,\partial)|\mathcal{F}(j;a)\rangle \tag{18}$$

with

$$\begin{aligned}\mathcal{H}_F(j,\partial) &:= j_{I_1}K_{I_1I_2}\partial_{I_2} + W_{I_1I_2I_3I_4}\\ &\cdot\big[j_{I_1}\partial_{I_4}\partial_{I_3}\partial_{I_2} - 3F_{I_4K}j_{I_1}j_K\partial_{I_3}\partial_{I_2}\\ &\quad +(3F_{I_4K_1}F_{I_3K_2}+\frac{1}{4}A_{I_4K_1}A_{I_3K_2})j_{I_1}j_{K_1}j_{K_2}\partial_{I_2}\\ &\quad -(F_{I_4K_1}F_{I_3K_2}+\frac{1}{4}A_{I_4K_1}A_{I_3K_2})\\ &\qquad\cdot F_{I_2K_3}j_{I_1}j_{K_1}j_{K_2}j_{K_3}\big].\end{aligned} \tag{19}$$

Equation (19) is the algebraic Schroedinger representation of the spinorfield Lagrangian (1), written in functional space with a fixed algebraic state space. With respect to the physical interpretation of the algebraic Schroedinger representation and the necessity of their application as well as their probability conservation we refer to the comments given in [15], see also [13].

3. Algebraic Representation of Effective Theories

Effective theories are generated if in the functional formulation of the algebraic Schroedinger representation of the spinor field (16), (18), (19) mappings on other appropriate functional spaces are performed. The mathematical foundation of this mapping formalism was developed in a previous paper [15]. Without going into details we refer to this paper and give only the final formulas for the case under consideration.

In the spinor field model it is assumed that electroweak gauge bosons of the Standard model possess a substructure consisting of two partons, while leptons and quarks are to be formed by three partons. Starting with the spinor field dynamics (19) the mapping of the combined two-parton-three-parton many particle system was studied and its effective dynamics formally derived. We will explicitly evaluate this representation for the case of CP-symmetry breaking.

Let the functional states of the corresponding effective field theory be defined by

$$|\mathcal{P}(b,f;a)\rangle := \sum_{n=2}^{\infty}\sum_{m=0}^{\infty}\sum_{r=0}^{\infty}\delta_{n,2m+3r}\frac{1}{(m!)(r!)}\tilde{\varrho}(k_1\ldots k_m,q_1\ldots q_r|a)b_{k_1}\ldots b_{k_m}f_{q_1}\ldots f_{q_r}|0\rangle_{BF}, \tag{20}$$

where b,∂^b and f,∂^f are the functional sources of the phenomenological bosons and fermions, respectively, while the $\tilde{\varrho}$-functions represent the matrix elements of the effective (phenomenological) boson-fermion theory in analogy to the definition of τ_n in (7), but not of φ_n in (16) as the effective theory is not normal ordered!

Then the mapping of equation (18) leads to an effective functional energy operator $\tilde{\mathcal{H}}$ (see [15], equation (82)) and its eigenvalue equation

$$E|\mathcal{P}(b,f)\rangle = \tilde{\mathcal{H}}(b,\partial^b,f,\partial^f)|\mathcal{P}(b,f)\rangle. \qquad (21)$$

The energy operator $\tilde{\mathcal{H}}$ can be decomposed into leading terms, higher order terms and quantization terms which result from the quantization terms in $\mathcal{H}_F$ by the mapping. As we are only interested in the dynamical structure of the effective theory, we exclude the quantization terms from our discussion.

With respect to the higher order terms, estimates were performed in [9] and [16]. In his thesis Fuss has developed a systematic calculation scheme for the estimate of these terms [17]. For heavy spinor field masses m_i in (1) these terms are tiny and can be omitted from the physical discussion, in particular in the low energy range. It would exceed the scope of this paper to discuss this calculation scheme and its estimates explicitly. So in this paper we treat only the leading terms which are physically relevant.

Decomposing $\tilde{\mathcal{H}}$ into

$$\tilde{\mathcal{H}} = \mathcal{H}_f + \mathcal{H}_b + \mathcal{H}_{bf} \qquad (22)$$

for the leading terms the following expressions result from the mapping theorems [15]:

$$\begin{aligned}
\mathcal{H}_f &:= K^f_{qp} f_q \partial^f_q + M^f_{qp} f_q \partial^f_p,\\
\mathcal{H}_b &:= K^b_{kl} b_k \partial^b_l + M^b_{kl} b_k \partial^b_l + W_1^{kl_1l_2} b_k \partial^b_{l_1} \partial^b_{l_2}, \qquad (23)\\
\mathcal{H}_{bf} &:= W_2^{qlp} \partial^b_l f_q \partial^f_p + W_4^{q_1q_2p_1p_2} R^k_{q_1q_2} b_k \partial^f_{p_1} \partial^f_{p_2},
\end{aligned}$$

with (see [15], equation (66))

$$\begin{aligned}
K^f_{qp} &:= 3R^q_{II'I_1} K_{I_1I_2} C^p_{I_2II'},\\
M^f_{qp} &:= -9W_{I_1I_2I_3I_4} F_{I_4K} (R^q_{IKI_1} C^p_{I_2I_3I}\\
&\qquad - R^q_{II'K} C^p_{I_2I_3I'}),\\
K^b_{kl} &:= 2R^k_{II_1} K_{I_1I_2} C^l_{I_2I},\\
M^b_{kl} &:= -6W_{I_1I_2I_3I_4} F_{I_4K} R^k_{KI_1} C^l_{I_2I_3}, \qquad (24)\\
W_1^{kl_1l_2} &:= 4W_{I_1I_2I_3I_4} R^k_{II_1} C^{l_1}_{I_4I} C^{l_2}_{I_2I_3},\\
W_2^{qlp} &:= 3W_{I_1I_2I_3I_4} R^q_{II'I_1} C^p_{I_4II'} C^l_{I_2I_3},\\
W_4^{q_1q_2p_1p_2} &:= -54W_{I_1I_2I_3I_4} F_{I_4K} R^{q_1}_{II'I_1}\\
&\qquad \cdot R^{q_2}_{K_1K_2K} C^{p_1}_{I'K_1K_2} C^{p_2}_{I_2I_3I}.
\end{aligned}$$

In these expressions the quantities $C^l_{II'}$ and $R^n_{II'}$ symbolize the boson states and their duals, while $C^q_{II'I''}$ and $R^p_{II'I''}$ are the fermion states and their duals both of which will be explicitly introduced in Section 4. The other quantities appearing in (24) are the terms of the original spinor field (11) – (14) and (17) which determine the structure and the numerical values of the effective theory defined by the energy operator (22).

The above mapping formalism is ideally suited for the derivation of effective theories: While the effective functional energy operator (22) represents the self energies and the direct interactions of the composite particles, the state functional (20) contains by definition of $\tilde{\varrho}$ the effect of the antisymmetrization of the whole set of composite particle states in all possible configurations in form of exchange integrals, see [15], equation (52), (53). Effective theories are then derived by the assumption that a physical situation is described where the effects of these exchange integrals can be neglected. I.e., in this approximation the composite particles feel the internal structure of their direct reaction partners, but not the embedding into the overall fermionic many particle system to which they belong.

This property can be used in the case of CP-symmetry breaking. According to [10], CP-symmetry breaking manifests itself in a violation of the permutation group representation, i.e., the occurence of parafermions in the boson functions and the appearance of additional boson states. The latter states can be incorporated into the set of composite particle functions, while the main effect of parafermions results in a modification of the exchange integrals in $\tilde{\varrho}$. By definition the latter effects are excluded from the derivation of the classical version of the effective theories, while owing to the modified particle representations the modified reaction channels appear in the effective theory as well as in its classical version. Thus as long as one does not intend to study the effect of exchange forces which is not the aim of the treatment given here, one can apply the above mapping formalism to the case of CP-symmetry breaking too.

4. Boson and Fermion Basis States

For the evaluation of the effective theory the states of the composite particles are required. These are two-parton bound states for the bosons whereas for fermions three-parton states are assumed. In order to get an optimal adaption to the structure of the physical particles these states should be derived from corresponding solutions of generalized de Broglie-Bargmann-Wigner (GBBW)-equations. For details see [18, 19].

For a first step in the investigation of the phenomena connected with CP-symmetry breaking we are mainly interested in the group theoretical structure of states and their effective dynamics. We thus confine ourselves to the use of test functions for the composite particle states which exactly fulfil all group theoretical requirements, but leave open the exact dependence on their space-time coordinates as far as it is not fixed by group theory. So the numerical calculation of effective physical constants by means of well defined space-time parts of the wave functions has to be done elsewhere.

For the bosons the exact vector boson state solutions of the GBBW-equations have been derived for the case of CP-symmetry breaking in a previous paper [10]. Hence for the introduction of appropriate test functions all group theoretical properties can be adopted from the exact solutions.

For the fermion states no exact state solutions of the corresponding GBBW-equations are known. Thus an idea how these states should be group theoretically modified by CP-symmetry breaking has to be borrowed from phenomenology.

In the Standard model the fermion mass eigenstates are determined by the Yukawa coupling to the Higgs fields. If several generations of fermions are taken into account then there is no reason for the fermion mass matrices of being diagonal. Indeed suitable mass matrices lead to CP-symmetry breaking effects. But, and this is very important: If the neutrinos are massless or nearly massless, then the CP-symmetry breaking terms cannot affect the lepton part of the mass matrix. Only the quark mass matrix is remarkably affected by these symmetry breaking terms ([20], p. 116, [21], p. 47).

This means: By CP-symmetry breaking only the group theoretical structure of the quark states will be remarkably modified compared with the states for unbroken CP-symmetry, whereas the lepton states are insensitive to this kind of symmetry breaking.

Owing to this obvious difference in the behavior of lepton and quark states under CP-symmetry breaking we confine ourselves to the treatment of the lepton states only as for these states the full group theoretical information of the CP-invariant theory can be used. Furthermore with regard to the very small but finite neutrino masses, we do without the introduction of right-handed and left-handed lepton states in order to simplify matters.

Before going into details of the definition of wave functions for the calculation of effective theories, it should be noted that exact solutions of GBBW-equations cannot be used as test functions themselves. Eigenfunctions are completely fixed in their functional form and their parameters. If, however, a composite particle is inserted into an assemblage of other particles, its internal structure must be adapted to the influence of this surrounding and the state of this particle can be no longer described as an eigensolution of the GBBW-equations. Therefore in deriving effective theories one is forced to consider test functions with freely variable parameters which can react on external forces [9, 14].

For the case of CP-symmetry breaking the exact solutions of the GBBW-equations for electroweak bosons are known. Their superspin-isospin part is given by a singlet and a triplet matrix representation and defined by the following sets of symmetric and antisymmetric matrices:

$$S^l = \begin{pmatrix} 0 & \sigma^l \\ (-1)^{l+1}\sigma^l & 0 \end{pmatrix}; \; T^l = \begin{pmatrix} 0 & \sigma^l \\ (-1)^l\sigma^l & 0 \end{pmatrix} \tag{25}$$

for the triplet, and

$$S^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}; \quad T^0 = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \tag{26}$$

for the singlet.

For convenience these superspin-isospin matrices are represented by Pauli matrices and by linear combinations of (25) the charge operator can be diagonalized. The representations (25), (26) are referred to the spinor basis (4). As later on a modified spinor basis will be introduced, we denote the basis (4) by the superscript S (superspinor)

For CP-symmetry breaking an eigenfunction reads in the basis (25), (26) (see [10])

$$\varphi^{\kappa_1\kappa_2}_{\alpha_1\alpha_2}(x_1, x_2|k, l)_{i_1 i_2} = (S^l + T^l)^S_{\kappa_1\kappa_2}\varphi_{\alpha_1\alpha_2}(x_1, x_2|k)_{i_1 i_2} \tag{27}$$

with

$$\varphi_{\alpha_1\alpha_2}(x_1, x_2|k, l)_{i_1 i_2} = \exp[-\mathrm{i}\frac{k}{2}(x_1 + x_2)]\big\{A^l_\mu\chi^\mu_{\alpha_1\alpha_2}(x_1 - x_2|k)_{i_1 i_2} + G^l_\mu\zeta^\mu_{\alpha_1\alpha_2}(x_1 - x_2|k)_{i_1 i_2}\big\}, \tag{28}$$

and with $p_+ := p + k/2$ and $p_- := p - k/2$ the following definitions hold:

$$\chi^\mu_{\alpha_1\alpha_2}(x|k)_{i_1i_2} := \frac{2\mathrm{i}g}{(2\pi)^4}\lambda_{i_1}\lambda_{i_2}\int \mathrm{d}^4p\ \exp(-\mathrm{i}px)[S_F(p_+, m_{i_1})\gamma^\mu S_F^{CP}(p_-, m_{i_2})C]_{\alpha_1\alpha_2} \tag{29}$$

and

$$\zeta^\mu_{\alpha_1\alpha_2}(x|k)_{i_1i_2} := \frac{2\mathrm{i}g}{(2\pi)^4}\lambda_{i_1}\lambda_{i_2}\int \mathrm{d}^4p\ \exp(-\mathrm{i}px)[S_F(p_+, m_{i_1})\gamma^5\gamma^\mu S_F^{CP}(p_-, m_{i_2})C]_{\alpha_1\alpha_2}, \tag{30}$$

where S_F^{CP} is the CP-symmetry breaking Feynman propagator.

For the spin orbit parts (28) it can be shown that A^l_μ and G^l_μ have to be identified with the phenomenological electric and magnetic vector potential of the boson state. In addition it is important to realize that in (28) the phenomenological electromagnetic and the linearized electroweak field strength tensors $F^{\ a}_{\mu\nu}$ occur too. These are terms of the kind

$$k_\nu A^a_\mu \Sigma^{\nu\mu} C \equiv F^a_{\nu\mu}\Sigma^{\nu\mu}C; \quad \varepsilon^{\mu\nu\varrho\lambda}k_\varrho G^a_\lambda \Sigma_{\nu\mu}C \equiv F^{\nu\mu}_a \Sigma_{\nu\mu}C, \tag{31}$$

which are contained in (28) (compare [9], p. 251) for the electric potential.

If for simplified test functions all terms which contain p_μ vectors are neglected ($\equiv$ s-wave approximation) then (28) can be replaced by the expression

$$\begin{aligned}\varphi_{\alpha_1\alpha_2}(x_1, x_2|k)_{i_1i_2} = \exp[-\mathrm{i}\frac{k}{2}(x_1+x_2)]\{&A^l_\mu(\gamma^\mu C)_{\alpha_1\alpha_2}\omega(x_1-x_2|k)_{i_1i_2}\\ &+ G^l_\mu(\gamma^5\gamma^\mu C)_{\alpha_1\alpha_2}\vartheta(x_1-x_2|k)_{i_1i_2} + F^l_{\nu\mu}(\Sigma^{\nu\mu}C)_{\alpha_1\alpha_2}\varrho(x_1-x_2|k)_{i_1i_2}\}.\end{aligned} \tag{32}$$

For solutions of the GBBW-equations the relations between the vectors A_μ, G_μ and the field strength tensor $F_{\mu\nu}$ are fixed. However, for test functions we consider the quantities A_μ, G_μ and $F_{\mu\nu}$ as unconstrained, freely variable quantities which can be adapted to interactions described in terms of the effective field equations. As a consequence the wave functions (27) have to be decomposed into three independent parts, associated to the field variables A_μ, G_μ and $F_{\mu\nu}$.

For the evaluation of the effective theory the single time wave functions are needed. For performing the transition to equal times we refer to [9] (p. 153). Owing to the translational invariance of the system we use the limit $t_1 = t_2 = 0$ without loss of generality. In this limit the functions $\omega, \vartheta, \varrho$ go over into f^A, f^G, f^F and the corresponding wave functions read

$$\begin{aligned}C^A_{Z_1Z_2}(\mathbf{r}_1, \mathbf{r}_2|\mathbf{k}, l, \mu) &:= (S^l+T^l)^S_{\kappa_1\kappa_2}\exp[-\mathrm{i}\frac{\mathbf{k}}{2}(\mathbf{r}_1+\mathbf{r}_2)](\gamma^\mu C)_{\alpha_1\alpha_2}f^A(\mathbf{r}_1-\mathbf{r}_2)_{i_1i_2},\\ C^G_{Z_1Z_2}(\mathbf{r}_1, \mathbf{r}_2|\mathbf{k}, l, \mu) &:= (S^l+T^l)^S_{\kappa_1\kappa_2}\exp[-\mathrm{i}\frac{\mathbf{k}}{2}(\mathbf{r}_1+\mathbf{r}_2)](\gamma^5\gamma^\mu C)_{\alpha_1\alpha_2}f^G(\mathbf{r}_1-\mathbf{r}_2)_{i_1i_2},\\ C^F_{Z_1Z_2}(\mathbf{r}_1, \mathbf{r}_2|\mathbf{k}, l, \mu, \nu) &:= (S^l+T^l)^S_{\kappa_1\kappa_2}\exp[-\mathrm{i}\frac{\mathbf{k}}{2}(\mathbf{r}_1+\mathbf{r}_2)](\Sigma^{\mu\nu} C)_{\alpha_1\alpha_2}f^F(\mathbf{r}_1-\mathbf{r}_2)_{i_1i_2},\end{aligned} \tag{33}$$

with $Z := (i, \alpha, \kappa)$. According to [22] the duals are defined by $R := \lambda^{-1}_{i_1}\lambda^{-1}_{i_2}C^+$ which need not to be explicitly represented here.

Concerning the fermion states, their group theoretical analysis has been performed in several papers for unbroken CP-symmetry [19, 22 – 25]. As was mentioned above the lepton states are insensitive to CP-symmetry breaking, i. e., one can apply the lepton states for conserved CP-symmetry.

For conserved symmetries the permutation group representations play an essential role in the construction of appropriate wave functions. For obtaining lepton states we thus adopt the group theoretical representations of test functions from Pfister [24], being based on the theory of representations of the permutation group elaborated by Kramer et. al. [26].

The group theoretical analysis of the three-parton problem must guarantee that the resulting test functions possess quantum numbers which coincide with those of the leptons in phenomenological theory. This can only be achieved by using mixed representations of the permutation group. Such mixed representations are generated by the application of Young operators C_{ik}. For two dimensional representations these operators are defined by the relations [24, 26]

$$
\begin{aligned}
C_{11}^{[21]} &:= \frac{1}{2}(1 - P_{12})\frac{1}{3}(2 + P_{13} + P_{23}),\\
C_{22}^{[21]} &:= \frac{1}{2}(1 + P_{12})\frac{1}{3}(2 - P_{13} - P_{23}),\\
C_{12}^{[21]} &:= \frac{1}{2}(1 - P_{12})\frac{\sqrt{3}}{3}(P_{23} - P_{13}),\\
C_{21}^{[21]} &:= \frac{1}{2}(1 + P_{12})\frac{\sqrt{3}}{3}(P_{23} - P_{13}),
\end{aligned} \tag{34}
$$

where P_{ik} means transposition which interchanges arguments with index i and k. These operators will be applied to superspin-isospin states and separately to spin-orbit states, see (35).

The use of the Young operators allows to start with products of test wave functions which are not antisymmetrized from the beginning. For lepton states these products have to be formed by superspin-isospin test-functions Θ^j and spin-orbit test functions $\Omega \otimes \psi$.

The superspin-isospin testfunctions are responsible for the definition of the phenomenological quantum numbers for isospin and charge and for the fermion number, while the spin-orbit test functions should lead to the spin 1/2 of the leptons, to generation numbers and (or) internal excitation levels. The latter two possibilities will not be further pursued in this investigation.

After rearrangements the general expression of Young combinations leads to the following antisymmetric test functions for leptons, [24], owing to $[21] \times [21] \to [111]$:

$$
\begin{aligned}
&C^{\kappa_1\kappa_2\kappa_3}_{\alpha_1\alpha_2\alpha_3}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3|\mathbf{k}, j, n) :=\\
&\exp[-\mathrm{i}\mathbf{k}\frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)][(C_{11}\Theta^j_{\kappa_1\kappa_2\kappa_3})\\
&\quad \cdot C_{22}\{\Omega^n_{\alpha_1\alpha_2\alpha_3}\psi(\mathbf{r}_2 - \mathbf{r}_1, \mathbf{r}_3 - \mathbf{r}_2)\}\\
&-(C_{21}\Theta^j_{\kappa_1\kappa_2\kappa_3})C_{12}\{\Omega^n_{\alpha_1\alpha_2\alpha_3}\psi(\mathbf{r}_2 - \mathbf{r}_1, \mathbf{r}_3 - \mathbf{r}_2)\}].
\end{aligned} \tag{35}
$$

A structurally transparent representation of the superspin-isospin parts in (35) can be derived if charge conjugated spinors are transformed into G-conjugated spinors by

$$
\psi^G_{\kappa\alpha i} = G_{\kappa\kappa'}\psi^C_{\kappa'\alpha i} \tag{36}
$$

with $G := \mathbf{1} \oplus c$ and $c = -i\sigma^2$.

In this representation characterized by the superscript D (decomposition), one decomposes the index κ into the double index (Λ, A) and obtains the following scheme:

$$
\begin{aligned}
(C_{11}\Theta^1)^{\mathrm{D}} &:= \chi^{1/2}_{1/2}(r_2) \otimes (1,1,1),\\
(C_{21}\Theta^1)^{\mathrm{D}} &:= \chi^{1/2}_{1/2}(r_1) \otimes (1,1,1) \equiv e^+,\\
(C_{11}\Theta^2)^{\mathrm{D}} &:= \chi^{1/2}_{-1/2}(r_2) \otimes (1,1,1),\\
(C_{21}\Theta^2)^{\mathrm{D}} &:= \chi^{1/2}_{-1/2}(r_1) \otimes (1,1,1) \equiv \bar{\nu}_{\mathrm{e}},\\
(C_{11}\Theta^3)^{\mathrm{D}} &:= \chi^{1/2}_{1/2}(r_2) \otimes (2,2,2),\\
(C_{21}\Theta^3)^{\mathrm{D}} &:= \chi^{1/2}_{1/2}(r_1) \otimes (2,2,2) \equiv \nu_{\mathrm{e}},\\
(C_{11}\Theta^4)^{\mathrm{D}} &:= \chi^{1/2}_{-1/2}(r_2) \otimes (2,2,2),\\
(C_{21}\Theta^4)^{\mathrm{D}} &:= \chi^{1/2}_{-1/2}(r_1) \otimes (2,2,2) \equiv e^-,
\end{aligned} \tag{37}
$$

with

$$
\begin{aligned}
\chi^{1/2}_{1/2}(r_1) &:= \left(\frac{2}{3}\right)^{1/2} \delta_{1A_1}\delta_{1A_2}\delta_{2A_3}\\
&\quad - \left(\frac{1}{6}\right)^{1/2} [\delta_{2A_1}\delta_{1A_2} + \delta_{1A_1}\delta_{2A_2}]\delta_{1A_3},\\
\chi^{1/2}_{1/2}(r_2) &:= \left(\frac{1}{2}\right)^{1/2} [\delta_{1A_1}\delta_{2A_2} - \delta_{2A_1}\delta_{1A_2}]\delta_{1A_3},\\
\chi^{1/2}_{-1/2}(r_1) &:= -\left(\frac{2}{3}\right)^{1/2} \delta_{2A_1}\delta_{2A_2}\delta_{1A_3}\\
&\quad + \left(\frac{1}{6}\right)^{1/2} [\delta_{2A_1}\delta_{1A_2} + \delta_{1A_1}\delta_{2A_2}]\delta_{2A_3},\\
\chi^{1/2}_{-1/2}(r_2) &:= \left(\frac{1}{2}\right)^{1/2} [\delta_{1A_1}\delta_{2A_2} - \delta_{2A_1}\delta_{1A_2}]\delta_{2A_3},\\
(1,1,1) &:= \delta_{1\Lambda_1}\delta_{1\Lambda_2}\delta_{1\Lambda_3};\ (2,2,2) := \delta_{2\Lambda_1}\delta_{2\Lambda_2}\delta_{2\Lambda_3}.
\end{aligned} \tag{38}
$$

The quantum numbers of these states coincide with the phenomenological quantum numbers (see [25], Table I), and in (37) the last column corresponds to the phenomenological spinor fields ψ^G afterwards. If the latter are transformed into the phenomenological S-representation by application of (36) the positions of ν and e^- are interchanged. This change is essential for the interpretation of the subsequent results, as (e^-, ν) is charge conjugated to $(e^+, \bar{\nu})$.

In defining the spin tensor we expect to obtain lepton fields $l^j_\alpha(x)$ in the effective theory which are *not* eigenstates of the Dirac operator for definite ***k***-vector. Hence we are not allowed to construct the spin tensor Ω by means of eigensolutions to ***k***-vectors.

Furthermore as the leptons are assumed to occupy the ground states of the three-parton system, the spin tensor as well as the orbit functions must show the highest possible invariance under symmetry operations, which for these parts of the wave functions are the little group operations with all discrete transformations. This leads to the spin tensor and its charge conjugated counterpart

$$\begin{aligned} \Omega^n_{\alpha_1\alpha_2\alpha_3} &= C_{\alpha_1\alpha_2}\xi^n_{\alpha_3}, \\ \bar{\Omega}^n_{\alpha_1\alpha_2\alpha_3} &= C_{\alpha_1\alpha_2}C_{\alpha_3\alpha}\xi^n_\alpha, \end{aligned} \tag{39}$$

where ξ^n_α are the four unit spinors $\delta_{\alpha n},\ n = 1, 2, 3, 4,$ while C is invariant under rotations and the discrete operation PC (see [27], p. 110). The orbit part is assumed to have s-wave character which automatically is invariant under parity transformations.

5. Effective Functional Energy Operator for Bosons

Before starting the calculations in this and the following sections it should be emphasized: All algebraic calculations of the superspin-isospin algebra and of the spin algebra are exact. But for the sake of brevity only the initial formulas and the final formulas will be explicitly reproduced in combination with some hints concerning their evaluation.

As far as boson states are concerned the corresponding calculations require gauge fixing. Although in principle the various gauges are assumed to be equivalent, a gauge fixing is necessary in order to adapt the general theory to the special calculation scheme. In case the algebraic Schroedinger representation is used, one has to apply a noncovariant gauge, as the latter should have the same invariance group as the former, i.e. the little group. In particular the temporal gauge is a distinguished gauge in this respect, as it is insensitive to symmetry breaking (compare [9], chapter 8).

In contrast to the electric vector potential for the magnetic vector potential the applicability of the temporal gauge has not yet been investigated so far. Nevertheless in the following we apply the temporal gauge to both kinds of vector potentials, the selfconsistency of which can be tested by examining the structure of the effective theory. Any incompatibility would manifest itself by a discrepancy in the structure of the resulting effective theory.

The effective functional boson energy operator is defined by $\mathcal{H}_b$ in (23). We treat the three terms of $\mathcal{H}_b$ in turn.

(i) The first term reads

$$\mathcal{H}^1_b := K^b_{kl} b_k \partial^b_l \equiv 2R^k_{II_1} K_{I_1I_2} C^l_{I_2I} b_k \partial^b_l. \tag{40}$$

The boson functions are defined by (33) and consequently the sources and their duals have to be characterized by the same indices. Substituting these functions into (40) one obtains with the definitions

$$\begin{aligned} f^A_{ii'}(\mathbf{r}|l)_{\alpha\alpha'} &:= f^A_{ii'}(\mathbf{r})(\gamma^l C)_{\alpha\alpha'}, \\ r^A_{ii'}(\mathbf{r}|l)_{\alpha\alpha'} &:= r^A_{ii'}(\mathbf{r})(\gamma^l C)^+_{\alpha\alpha'}, \\ f^G_{ii'}(\mathbf{r}|m)_{\alpha\alpha'} &:= f^G_{ii'}(\mathbf{r})(\gamma^5\gamma^m C)_{\alpha\alpha'}, \\ r^G_{ii'}(\mathbf{r}|m)_{\alpha\alpha'} &:= r^G_{ii'}(\mathbf{r})(\gamma^5\gamma^m C)^+_{\alpha\alpha'}, \\ f^F_{ii'}(\mathbf{r}|\mu\nu)_{\alpha\alpha'} &:= f^F_{ii'}(\mathbf{r})(\Sigma^{\mu\nu} C)_{\alpha\alpha'}, \\ r^F_{ii'}(\mathbf{r}|\mu\nu)_{\alpha\alpha'} &:= r^F_{ii'}(\mathbf{r})(\Sigma^{\mu\nu} C)^+_{\alpha\alpha'}, \end{aligned} \tag{41}$$

the explicit representation of (40) in the form

$$\begin{aligned} \mathcal{H}^1_b := \int & \mathrm{d}^3k\, \mathrm{d}^3k'\, \mathrm{d}^3r_1\, \mathrm{d}^3r_2\, (T^a + S^a)^+_{\kappa_1\kappa}(T^{a'} + S^{a'})_{\kappa\kappa_1} \exp[-\mathrm{i}\mathbf{k}\frac{1}{2}(\mathbf{r}+\mathbf{r}_1)][r^A_{ii_1}(\mathbf{r}-\mathbf{r}_1|l)_{\alpha\alpha_1} b^A_{l,a}(\mathbf{k}) \\ &+ r^F_{ii_1}(\mathbf{r}-\mathbf{r}_1|l,\nu)_{\alpha\alpha_1} b^F_{l,\nu,a}(\mathbf{k}) \\ &+ r^G_{ii_1}(\mathbf{r}-\mathbf{r}_1|l)_{\alpha\alpha_1} b^G_{l,a}(\mathbf{k})]2[-\mathrm{i}(\gamma^0\gamma^k)_{\alpha_1\alpha_2}\partial_1 + m_i\gamma^0_{\alpha_1\alpha_2}] \exp[\mathrm{i}\mathbf{k}'\frac{1}{2}(\mathbf{r}_1+\mathbf{r})] \\ &\cdot [f^A_{ii_1}(\mathbf{r}_1-\mathbf{r}|m)_{\alpha_2\alpha}\partial^A_{m,a'}(\mathbf{k}') + f^F_{ii_1}(\mathbf{r}_1-\mathbf{r}|m,\varrho)_{\alpha_2\alpha}\partial^F_{m,\varrho,a'}(\mathbf{k}') + f^G_{ii_1}(\mathbf{r}_1-\mathbf{r}|m)_{\alpha_2\alpha}\partial^G_{m,a'}(\mathbf{k}')]. \end{aligned} \tag{42}$$

It is convenient to pass from the functional generators b^F for the field tensor to functional generators b^E and b^B for the electric and magnetic fields by the decomposition

$$(\Sigma^{l\nu}C)b^F_{l\nu} = (\Sigma^{0m}C)b^E_m + (\Sigma^m C)b^B_m \tag{43}$$

with $b_m^B := -(1/2)\sum_{ij}\varepsilon_{ijm}b_{ij}^F$ and corresponding definitions for ∂_m^E and ∂_m^B. If this decomposition is substituted in (42), then after introduction of center of mass coordinates and algebraic conversions the following scalarproducts in the internal coordinate $\mathbf{u}$ result:

$$\begin{aligned}&\langle r_{ii'}^A f_{ii'}^G\rangle = \langle r_{ii'}^G f_{ii'}^A\rangle = c_1,\ \langle r_{ii'}^A m_i f_{ii'}^E\rangle = \langle r_{ii'}^E m_i f_{ii'}^A\rangle = c_2,\\ &\langle r_{ii'}^G m_i f_{ii'}^B\rangle = \langle r_{ii'}^B m_i f_{ii'}^G\rangle = c_3,\ \langle r_{ii'}^E f_{ii'}^B\rangle = \langle r_{ii'}^B f_{ii'}^E\rangle = 1,\end{aligned} \tag{44}$$

if one observes the definition of the duals. Then one obtains for (42)

$$\begin{aligned}\mathcal{H}_b^1 = &\int \mathrm{d}^3z\, b_{la}^A(\mathbf{z})[c_1\varepsilon_{klm}\partial_k^z\partial_{ma}^G(\mathbf{z}) + \mathrm{i}c_2\partial_{la}^E(\mathbf{z})] + \int \mathrm{d}^3z\, b_{la}^G(\mathbf{z})[c_1\varepsilon_{klm}\partial_k^z\partial_{ma}^A(\mathbf{z}) + c_3\partial_{la}^B(\mathbf{z})]\\ &+ \mathrm{i}\int \mathrm{d}^3z\, b_{la}^E(\mathbf{z})[\varepsilon_{klm}\partial_k^z\partial_{ma}^B(\mathbf{z}) - c_2\partial_{la}^A(\mathbf{z})] - \mathrm{i}\int \mathrm{d}^3z\, b_{la}^B(\mathbf{z})[\varepsilon_{klm}\partial_k^z\partial_{ma}^E(\mathbf{z}) + \mathrm{i}c_3\partial_{la}^G(\mathbf{z})],\end{aligned} \tag{45}$$

where the spin algebra multiplication tables in [27] (section 4.3) have been used.

(ii) The second term of $\mathcal{H}_b$ in (23) reads

$$\mathcal{H}_b^2 := M_{kl}^b b_k\partial_l^b := -6W_{I_1I_2I_3I_4}F_{I_4K}R_{KI_1}^k C_{I_2I_3}^l b_k\partial_l^b. \tag{46}$$

It contains the vertex (13), (14) with three δ-distributions. In representing this term, we reproduce it in the form where the integrations over the coordinates of the δ-distributions have already been carried out. Then one gets the following expression:

$$\begin{aligned}\mathcal{H}_b^2 = g\int &\mathrm{d}^3r_1\,\mathrm{d}^3r\,\mathrm{d}^3k\,\mathrm{d}^3k'\,\Big\{\lambda_{i_1}\sum_h[(\gamma^0v^h)_{\beta_1\beta_2}(v^hC)_{\beta_3\beta_4}\delta_{\varrho_1\varrho_2}\gamma^5_{\varrho_3\varrho_4} - (\gamma^0v^h)_{\beta_1\beta_3}(v^hC)_{\beta_2\beta_4}\delta_{\varrho_1\varrho_3}\gamma^5_{\varrho_2\varrho_4}\\ &- (\gamma^0v^h)_{\beta_1\beta_4}(v^hC)_{\beta_3\beta_2}\delta_{\varrho_1\varrho_4}\gamma^5_{\varrho_3\varrho_2}]\Big\}\sum_{i_4}\lambda_{i_4}\gamma^5_{\varrho_4\kappa}\delta_{i_4i'}\big[g_{i_4}(\mathbf{r}_1-\mathbf{r})C_{\beta_4\alpha} + g_{i_4}^k(\mathbf{r}_1-\mathbf{r})(\gamma^kC)_{\beta_4\alpha}\big]\\ &\cdot(T^{a'}+S^{a'})_{\varrho_2\varrho_3}\big[\hat f^A(0|m)_{\beta_2\beta_3}\partial_{ma'}^A(\mathbf{k}') + \hat f^E(0|m)_{\beta_2\beta_3}\partial_{ma'}^E(\mathbf{k}') + \hat f^B(0|m)_{\beta_2\beta_3}\partial_{ma'}^B(\mathbf{k}')\\ &+ \hat f^G(0|m)_{\beta_2\beta_3}\partial_{ma'}^G(\mathbf{k}')\big]\exp[-\mathrm{i}\mathbf{k}'\mathbf{r}_1](T^a+S^a)^+_{\kappa\varrho_1}\big[r_{i_1i'}^A(\mathbf{r}_1-\mathbf{r}|l)_{\alpha\beta_1}b_{la}^A(\mathbf{k})\\ &+ r_{i_1i'}^E(\mathbf{r}_1-\mathbf{r}|l)_{\alpha\beta_1}b_{la}^E(\mathbf{k}) + r_{i_1i'}^B(\mathbf{r}_1-\mathbf{r}|l)_{\alpha\beta_1}b_{la}^B(\mathbf{k}) + r_{i_1i'}^G(\mathbf{r}_1-\mathbf{r}|l)_{\alpha\beta_1}b_{la}^G(\mathbf{k})\big]\exp[\mathrm{i}\mathbf{k}\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r})],\end{aligned} \tag{47}$$

where the definition (41) for the field tensor functions are transferred to the functions for the electric and magnetic fields and corresponding algebra elements according to (43). The second line of (47) contains a formal representation of the propagator.

Because the main effect of CP-symmetry breaking manifests itself in the emergence of parafermions (parapartons) which break the antisymmetry of the bosonic wave functions, and because this effect is included in the definitions of the wave functions (27)–(32), we apply the CP-symmetric propagator for the calculation of (47). Concerning its single time limit in terms of the formal representation given above we refer to [9], equation (7.79).

The omission of the CP-symmetry breaking part of the propagator in (47) enforces the electric and magnetic bosons to have equal masses in the effective theory which owing to the smallness of the symmetry breaking part is a good approximation.

If (47) is evaluated, the third term of the algebraic part of the vertex drops out, and the same holds for the terms connected with ∂^E and ∂^B. Furthermore the scalar product of the dual functions with the vector part of the propagator vanishes and in the exponential $\exp[-\mathrm{i}\mathbf{k}\mathbf{r}_1]$ with $\mathbf{r}_1 = \mathbf{z}+1/2\mathbf{u}$, the $\mathbf{u}$-part is neglected owing to the strong concentration of the dual functions around the origin of $\mathbf{u}$. One finally obtains

$$\begin{aligned}\mathcal{H}_b^2 = \mathrm{i}&\int \mathrm{d}^3z\hat f^A c_4 b_{la}^E(\mathbf{z})\partial_{la}^A(\mathbf{z})\\ &-\int \mathrm{d}^3z\hat f^G c_4 b_{la}^B(\mathbf{z})\partial_{la}^G(\mathbf{z})\end{aligned} \tag{48}$$

with

$$c_4 = \langle r^E_{i_1 i} g_i \rangle \lambda_i \lambda_{i_1} g \equiv \langle r^B_{i_1 i} g_i \rangle \lambda_i \lambda_{i_1} g. \tag{49}$$

Due to the intrinsic regularization all scalar products and constants are finite.

(iii) The third term reads

$$\begin{aligned}\mathcal{H}^3_b &:= W_1^{lmk} b_l \partial^b_m \partial^b_k \\ &= 4 W_{I_1 I_2 I_3 I_4} R^l_{I I_1} C^m_{I_4 I} C^k_{I_2 I_3} b_l \partial^b_m \partial^b_k.\end{aligned} \tag{50}$$

A comparison with (46) shows: In both terms the vertex is projected on $C^{\,k}_{I_2 I_3}$ in the same way. Hence we can adopt the evaluation of this part from (46), i. e., the third term of the algebraic part of the vertex drops out and the same holds for the electric and magnetic field terms of $C^{\,k}_{I_2 I_3}$. If in addition the three δ-distributions of the vertex are eliminated by integrations, one eventually obtains

$$\begin{aligned}\mathcal{H}^3_b = 4\lambda_{i_1} \int & \mathrm{d}^3 r_1\, \mathrm{d}^3 r\, \mathrm{d}^3 k\, \mathrm{d}^3 k'\, \mathrm{d}^3 k'' \Big\{ \sum_h \big[(\gamma^0 v^h)_{\beta_1 \beta_2} (v^h C)_{\beta_3 \beta_4} \delta_{\varrho_1 \varrho_2} \gamma^5_{\varrho_3 \varrho_4} \\ & - (\gamma^0 v^h)_{\beta_1 \beta_3} (v^h C)_{\beta_2 \beta_4} \delta_{\varrho_1 \varrho_3} \gamma^5_{\varrho_2 \varrho_4} \big] \Big\} (T^a + S^a)^+_{\varrho \varrho_1} \Big[r^A_{i i_1} (\mathbf{r} - \mathbf{r}_1 | l)_{\beta \beta_1} b^A_{l,a}(\mathbf{k}) \\ & + r^E_{i i_1} (\mathbf{r} - \mathbf{r}_1 | l)_{\beta \beta_1} b^E_{l,a}(\mathbf{k}) + r^B_{i i_1} (\mathbf{r} - \mathbf{r}_1 | l)_{\beta \beta_1} b^B_{l,a}(\mathbf{k}) + r^G_{i i_1} (\mathbf{r} - \mathbf{r}_1 | l)_{\beta \beta_1} b^G_{l,a}(\mathbf{k}) \Big] \\ & \cdot\, \exp[\mathrm{i}\mathbf{k} \tfrac{1}{2} (\mathbf{r} + \mathbf{r}_1)] (T^b + S^b)_{\varrho_2 \varrho_3} \big[\hat{f}^A (0|k)_{\beta_2 \beta_3} \partial^A_{m,b}(\mathbf{k}') + \hat{f}^G (0|k)_{\beta_2 \beta_3} \partial^G_{k,b}(\mathbf{k}') \big] \\ & \cdot\, \exp[-\mathrm{i}\mathbf{k}' \mathbf{r}_1] (T^c + S^c)_{\varrho_4 \varrho} \Big[f^A_{i_4 i} (\mathbf{r}_1 - \mathbf{r} | m)_{\beta_4 \beta} \partial^A_{m,c}(\mathbf{k}'') + f^E_{i_4 i} (\mathbf{r}_1 - \mathbf{r} | m)_{\beta_4 \beta} \partial^E_{m,c}(\mathbf{k}'') \\ & + f^B_{i_4 i} (\mathbf{r}_1 - \mathbf{r} | m)_{\beta_4 \beta} \partial^B_{m,c}(\mathbf{k}'') + f^G_{i_4 i} (\mathbf{r}_1 - \mathbf{r} | m)_{\beta_4 \beta} \partial^G_{m,c}(\mathbf{k}'') \Big] \exp[-\mathrm{i}\mathbf{k}'' \tfrac{1}{2} (\mathbf{r}_1 + \mathbf{r})].\end{aligned} \tag{51}$$

To understand the physical meaning of this formula we compare the superspin-isospin part of the CP-invariant effective gauge theory with that of the effective theory with CP-symmetry breaking. From the CP-invariant theory one gets $\mathrm{Tr}[(T^a)^+ (T^b \gamma^5) T^c]$ for the contribution of the superspin-isospin part to $\mathcal{H}^3_b$, while from (51) to $\mathcal{H}^3_b$ the superspin-isospin contribution $\mathrm{Tr}[(T^a + S^a)^+ (T^b \gamma^5)(T^c + S^c)]$ results.

This difference in these contributions leads to different effective theories. In the case of the CP-invariant theory the $SU(2)$-triplet and the $SU(2)$-singlet are completely decoupled, and the singlet contribution to $\mathcal{H}^3_b$ vanishes which forms the basis for the derivation of a $SU(2) \otimes U(1)$ effective gauge theory. In contrast to this result in the CP-symmetry violating theory one gets from (51) for $a, b, c = 0, 1, 2, 3$

$$\begin{aligned}&\mathrm{Tr}[(T^a + S^a)^+ (T^b \gamma^5)(T^c + S^c)] \\ &= \mathrm{Tr}[(T^a + S^a)^+ (S^b \gamma^5)(T^c + S^c)] \\ &= \mathrm{Tr}_2 [\sigma^a \sigma^b \sigma^c],\end{aligned} \tag{52}$$

if with σ^0 the unit matrix in the Pauli-algebra is denoted. The trace in two-dimensional isospin space yields ([27], p. 246)

$$\begin{aligned}\mathrm{Tr}_2 [\sigma^a \sigma^b \sigma^c] &= 2\mathrm{i} \varepsilon_{abc} + 2\delta_{ab} \delta_{0c} + 2\delta_{0a} \delta_{bc} \\ &\quad + 2\delta_{0b} \delta_{ac} =: \eta_{abc},\end{aligned} \tag{53}$$

where in the $\boldsymbol{\varepsilon}$-tensor the arguments are not allowed to take the value 0. Thus in this expression $SU(2)$ triplet and singlet states are mixed and as a consequence one cannot derive a $SU(2) \otimes U(1)$ invariant gauge theory, i. e., CP-symmetry breaking induces $SU(2)$ symmetry breaking.

As the temporal gauge can be applied in the case of symmetry breaking too, we use it for the evaluation of (51) in accordance with the treatment of the other terms. With the definitions

$$\begin{aligned}\sum_{i_4 i_1 i} g \lambda_{i_1} \langle r^A_{i i_1} f^G_{i_4 i} \rangle &=: \overline{\langle r^A f^G \rangle} = k_1, \\ \overline{\langle r^E f^B \rangle} &= \overline{\langle r^B f^E \rangle} =: k_2, \\ \overline{\langle r^G f^A \rangle} &=: k_3, \\ \overline{\langle r^A f^A \rangle} &=: k_4, \\ \overline{\langle r^E f^E \rangle} &= \overline{\langle r^B f^B \rangle} =: k_5, \\ \overline{\langle r^G f^G \rangle} &=: k_6,\end{aligned} \tag{54}$$

and with (52), (53) and the evaluation of the spin algebra, one obtains the following final form of (51):

$$\begin{aligned}\mathcal{H}^3_b = \eta_{abc} \varepsilon_{klm} \Big\{ & - 64 \hat{f}^A \\ & \cdot \int \mathrm{d}^3 z \big[\mathrm{i} k_1 b^A_{l,a}(\mathbf{z}) \partial^A_{k,b}(\mathbf{z}) \partial^G_{m,c}(\mathbf{z})\end{aligned}$$

$$
\begin{aligned}
&- k_2 b^E_{l,a}(\mathbf{z})\partial^A_{k,b}(\mathbf{z})\partial^B_{m,c}(\mathbf{z})\\
&+ k_2 b^B_{l,a}(\mathbf{z})\partial^A_{k,b}(\mathbf{z})\partial^E_{m,c}(\mathbf{z})\\
&+ ik_3 b^G_{l,a}(\mathbf{z})\partial^A_{k,b}(\mathbf{z})\partial^A_{m,c}(\mathbf{z})\big]\\
-64\hat f^G \int \mathrm{d}^3 z \big[&\mathrm{i}k_4 b^A_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^A_{m,c}(\mathbf{z})\\
&+ \mathrm{i}k_5 b^E_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^E_{m,c}(\mathbf{z})\\
&+ \mathrm{i}k_5 b^B_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^B_{m,c}(\mathbf{z})\\
&+ \mathrm{i}k_6 b^G_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^G_{m,c}(\mathbf{z})\big]\Big\}.
\end{aligned} \tag{55}
$$

In the absence of magnetic vector bosons (55) goes over into

$$
\begin{aligned}
\mathcal{H}^3_b = \mathrm{i}\varepsilon_{abc}\varepsilon_{klm} 128 \hat f^A k_2 &\\
\cdot \int \mathrm{d}^3 z \big[& b^E_{l,a}(\mathbf{z})\partial^A_{k,b}(\mathbf{z})\partial^B_{m,c}(\mathbf{z})\\
&- b^B_{l,a}(\mathbf{z})\partial^A_{k,b}(\mathbf{z})\partial^E_{m,c}(\mathbf{z})\big].
\end{aligned} \tag{56}
$$

This expression corresponds to the functional version of the nonlinear terms of an effective $SU(2)$-gauge theory (see [9], equation (7.101)). Conversely in the absence of electric vector bosons the corresponding expression does not lead to a gauge invariant theory. This result coincides with the phenomenological finding that nonabelian gauge theories possess no dual theory in contrast to the abelian Maxwell theory [28].

6. Effective Coupling of Bosons to Fermions

The corresponding term should lead to the bosonic vector potential in the fermionic equations of the effective theory. As the detailed evaluation will show this term has to be identified with the first term of $\mathcal{H}_{bf}$ in (23) defined by

$$
\begin{aligned}
\mathcal{H}^1_{bf} &= W_2^{kqp}\partial^b_k f_q \partial^f_p\\
&= 3W_{I_1I_2I_3I_4} R^q_{II'I_1} C^p_{I_4II'} C^k_{I_2I_3}\partial^b_k f_q \partial^f_p .
\end{aligned} \tag{57}
$$

If the treatment of (47) is used the following equation holds:

$$
\begin{aligned}
W_{I_1I_2I_3I_4} C^k_{I_2I_3}\partial^b_k = &\sum_{\mathbf{k}}\sum_{I_2I_3}\lambda_{i_1} B_{i_2i_3i_4}\int \mathrm{d}^3 r_2\,\mathrm{d}^3 r_3\,\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_1-\mathbf{r}_4)\\
&\cdot\sum_h \big[(\gamma^0 v^h)_{\beta_1\beta_2}(v^h C)_{\beta_3\beta_4}\delta_{\varrho_1\varrho_2}\gamma^5_{\varrho_3\varrho_4} - (\gamma^0 v^h)_{\beta_1\beta_3}(v^h C)_{\beta_2\beta_4}\delta_{\varrho_1\varrho_3}\gamma^5_{\varrho_2\varrho_4}\big](T^b+S^b)_{\varrho_2\varrho_3}\\
&\cdot\big[f^A_{i_2i_3}(\mathbf{r}_2-\mathbf{r}_3)(\gamma^k C)_{\beta_2\beta_3}\partial^A_{k,b}(\mathbf{k}) + f^G_{i_2i_3}(\mathbf{r}_2-\mathbf{r}_3)(\gamma^k\gamma^5 C)_{\beta_2\beta_3}\partial^G_{k,b}(\mathbf{k})\big]\exp[-\mathrm{i}\mathbf{k}\tfrac{1}{2}(\mathbf{r}_2+\mathbf{r}_3)],
\end{aligned} \tag{58}
$$

which leads to

$$
\begin{aligned}
W_{I_1I_2I_3I_4} C^k_{I_2I_3}\partial^b_k = -4\big[&(\gamma^0\gamma^k)_{\beta_1\beta_4}(T^b\gamma^5)_{\varrho_1\varrho_4}\hat f^A\partial^A_{k,b}(\mathbf{k})\\
&+ (\gamma^0\gamma^k\gamma^5)_{\beta_1\beta_2}(S^b\gamma^5)_{\varrho_1\varrho_4}\hat f^G\partial^G_{k,b}(\mathbf{k})\big]\lambda_{i_1} B_{i_4}\delta(\mathbf{r}_1-\mathbf{r}_4)\exp[-\mathrm{i}\mathbf{k}\mathbf{r}_1].
\end{aligned} \tag{59}
$$

We substitute this result into (57), integrate over $\mathbf{r}_4$ and rename $\mathbf{r}_1$ to $\mathbf{r}''$. Then we get

$$
\begin{aligned}
\mathcal{H}^1_{bf} = -12\sum_{\mathbf{k}\mathbf{k}'\mathbf{k}''}&\big[(\gamma^0\gamma^k)_{\beta_1\beta_4}(T^b\gamma^5)_{\varrho_1\varrho_4}\hat f^A\partial^A_{k,b}(\mathbf{k}) + (\gamma^0\gamma^k\gamma^5)_{\beta_1\beta_4}(S^b\gamma^5)_{\varrho_1\varrho_4}\hat f^G\partial^G_{k,b}(\mathbf{k})\big]\\
&\cdot\sum_{i_1i_4i}\lambda_{i_1}\int \mathrm{d}^3 r\,\mathrm{d}^3 r'\,\mathrm{d}^3 r''\, r^{\varrho\varrho'\varrho_1}_{\beta\beta'\beta_1}(\mathbf{r},\mathbf{r}',\mathbf{r}''|l,n)_{ii'i_1}\exp[\mathrm{i}\mathbf{k}'\tfrac{1}{3}(\mathbf{r}+\mathbf{r}'+\mathbf{r}'')]\\
&\cdot c^{\varrho_4\varrho\varrho'}_{\beta_4\beta\beta'}(\mathbf{r}'',\mathbf{r},\mathbf{r}'|j,m)_{ii'i_4}\exp[-\mathrm{i}\mathbf{k}''\tfrac{1}{3}(\mathbf{r}+\mathbf{r}'+\mathbf{r}'')]\exp[-\mathrm{i}\mathbf{k}\mathbf{r}'']f_{ln}(\mathbf{k}')\partial^f_{jm}(\mathbf{k}''),
\end{aligned} \tag{60}
$$

with l, j as superspin-isospin state numbers, and n, m as spin state numbers.

Introduction of center of mass coordinates

$$
z = \frac{1}{3}(\mathbf{r}+\mathbf{r}'+\mathbf{r}''),\quad \mathbf{u} = \mathbf{r}'-\mathbf{r},\quad \mathbf{v} = \mathbf{r}''-\mathbf{r}' \tag{61}
$$

and

$$
\mathbf{r} = \mathbf{z} - \frac{2}{3}\mathbf{u} - \frac{1}{3}\mathbf{v},\quad \mathbf{r}' = \mathbf{z} + \frac{1}{3}\mathbf{u} - \frac{1}{3}\mathbf{v},\quad \mathbf{r}'' = \mathbf{z} + \frac{1}{3}\mathbf{u} + \frac{2}{3}\mathbf{v} \tag{62}
$$

yields

$$\mathcal{H}^1_{bf} = -12\big[(\gamma^0\gamma^k)_{\beta_1\beta_4}(T^b\gamma^5)_{\varrho_1\varrho_4}\hat{f}^A\partial^A_{k,b}(\mathbf{z}) + (\gamma^0\gamma^k\gamma^5)_{\beta_1\beta_4}(S^b\gamma^5)_{\varrho_1\varrho_4}\hat{f}^G\partial^G_{k,b}(\mathbf{z})\big] \cdot \sum_{i_1i_4i}\lambda_{i_1}\int \mathrm{d}^3z\,\mathrm{d}^3u\,\mathrm{d}^3v\, r^{\varrho\varrho'\varrho_1}_{\beta\beta'\beta_1}(\mathbf{u},\mathbf{v}|l,n)_{ii'i_1} c^{\varrho\varrho'\varrho_4}_{\beta\beta'\beta_4}(\mathbf{u},\mathbf{v}|j,m)_{ii'i_4} f_{ln}(\mathbf{z})\partial^f_{jm}(\mathbf{z}), \tag{63}$$

where, owing to the strongly concentrated wave functions in $\mathbf{u}$ and $\mathbf{v}$ around the origin, the $\mathbf{u}$ and $\mathbf{v}$-terms in the exponentials have been neglected.

The further evaluation depends upon the fermionic wave functions which are defined in their general form by (35) and (36). In particular with (35) and with

$$Y^s \in \{(T^a\gamma^5),(S^a\gamma^5), a=0,1,2,3\}, \quad X^t \in \{(\gamma^0\gamma^k),(\gamma^0\gamma^k\gamma^5), k=1,2,3\} \tag{64}$$

the parts containing the wave functions in (63) can be written as follows:

$$r^{\varrho\varrho'\varrho_1}_{\beta\beta'\beta_1}(\mathbf{u},\mathbf{v}|ln) Y^s_{\varrho_1\varrho_4} X^t_{\beta_1\beta_4} c^{\varrho\varrho'\varrho_4}_{\beta\beta'\beta_4}(\mathbf{u},\mathbf{v}|jm) \equiv \big\{(C_{11}\Theta^l)_{\varrho\varrho'\varrho_1} C_{22}[\bar{\Omega}^n_{\beta\beta'\beta_1}\psi^*(\mathbf{r}_2-\mathbf{r}_1,\mathbf{r}_3-\mathbf{r}_2)] - (C_{21}\Theta^l)_{\varrho\varrho'\varrho_1} C_{12}[\bar{\Omega}^n_{\beta\beta'\beta_1}\psi^*(\mathbf{r}_2-\mathbf{r}_1,\mathbf{r}_3-\mathbf{r}_2)]\big\} Y^s_{\varrho_1\varrho_4} X^t_{\beta_1\beta_4} \big\{(C_{11}\Theta^j)_{\varrho\varrho'\varrho_4} C_{22}[\Omega^m_{\beta\beta'\beta_4}\psi(\mathbf{r}_2-\mathbf{r}_1,\mathbf{r}_3-\mathbf{r}_2)] - (C_{21}\Theta^j)_{\varrho\varrho'\varrho_4} C_{12}[\Omega^m_{\beta\beta'\beta_4}\psi(\mathbf{r}_2-\mathbf{r}_1,\mathbf{r}_3-\mathbf{r}_2)]\big\}. \tag{65}$$

The summations in (65) can be directly and explicitly performed for the various elements of (64). Using the definition

$$\langle\varphi'|A_{(3)}|\varphi\rangle := \sum_{i_1i_2i_3i_4}\varphi'(i_1,i_2,i_3)A_{i_3i_4}\varphi(i_1,i_2,i_4) \tag{66}$$

one first realizes that in C(≡S)- as well as in G(≡D)-representation

$$\langle C_{11}\Theta^l|Y^s_{(3)}|C_{21}\Theta^j\rangle = \langle C_{21}\Theta^l|Y^s_{(3)}|C_{11}\Theta^j\rangle = 0, \quad \forall\, s \tag{67}$$

holds. Furthermore direct calculation yields

$$\langle C_{11}\Theta^l|Y^s_{(3)}|C_{11}\Theta^j\rangle = Y^s_{lj} \quad \forall\, s, \tag{68}$$

and in both types of representation it follows

$$\langle C_{21}\Theta^l|(T^0\gamma^5)_{(3)}|C_{21}\Theta^j\rangle = (T^0\gamma^5)_{lj}, \quad \langle C_{21}\Theta^l|(T^a\gamma^5)_{(3)}|C_{21}\Theta^j\rangle = -\frac{1}{3}(T^a\gamma^5)_{lj}, \quad a=1,2,3. \tag{69}$$

The same relations hold if $(T^a\gamma^5)$ is replaced by $(S^a\gamma^5)$.

Concerning the spin orbit parts, owing to (67) only the combinations $C_{22}\times C_{22}$ and $C_{12}\times C_{12}$ have to be calculated. We substitute (39) into (65). For convenience we use the charge conjugated spin functions as both sets are equivalent. Then the calculation of $C_{12}\times C_{12}$ leads to

$$\langle C_{12}\bar{\Omega}^n\psi|X^t_{(3)}|C_{12}\Omega^m\psi\rangle = \xi^n_\alpha X^t_{\alpha\beta}\xi^m_\beta 3\left(\frac{1}{6}\right)^2\Big\{\psi^*(\mathbf{u}+\mathbf{v},-\mathbf{v})[\psi(-\mathbf{v},-\mathbf{u})+\psi(\mathbf{v},-\mathbf{u}-\mathbf{v})] + \psi^*(-\mathbf{v},-\mathbf{u})[\psi(\mathbf{u}+\mathbf{v},-\mathbf{v})+\psi(-\mathbf{u}-\mathbf{v},\mathbf{u})] + \psi^*(\mathbf{v},-\mathbf{u}-\mathbf{v})[\psi(\mathbf{u}+\mathbf{v},-\mathbf{v})+\psi(-\mathbf{u}-\mathbf{v},\mathbf{u})] + \psi^*(-\mathbf{u}-\mathbf{v},\mathbf{u})[\psi(-\mathbf{v},-\mathbf{u})+\psi(\mathbf{v},-\mathbf{u}-\mathbf{v})]\Big\}. \tag{70}$$

It should be noted that there exists still an additional set of terms which contains the traces of $X_{\alpha\beta}$ and thus vanishes owing to the special elements (64). Furthermore expression (70) does not vanish at the origin $\mathbf{u}=\mathbf{v}=0$, where one gets

$$\langle C_{12}\bar{\Omega}^n\psi|X^t_{(3)}|C_{12}\Omega^m\psi\rangle = -\left(\frac{1}{3}\right)\xi^n_\alpha X^t_{\alpha\beta}\xi^m_\beta|\psi(0,0)|^2 \tag{71}$$

with nonvanishing $\psi(0,0)$ due to its groundstate property.

In contrast to this result it is

$$\langle C_{22}\bar{\Omega}^n\psi|X^t_{(3)}|C_{22}\Omega^m\psi\rangle = \xi^n_\alpha X^t_{\alpha\beta}\xi^m_\beta \Psi(\mathbf{u},\mathbf{v}), \tag{72}$$

where the extensive expression Ψ cancels out at the origin.

Thus the matrix elements (70) or (71), respectively, must be considered as the leading terms of (65), as owing to the concentration of the wave functions ψ around the origin, the most important contribution to the values of these matrix elements comes from the value at this place. From (68), (69) and (70), (72) it follows that the two terms have the same group theoretical structure. This fact allows to neglect the matrix elements (72) and to concentrate on the leading term (70) as the omission of (72) from the general expression (65) does not alter its group theoretical structure which is crucial for the effective theory. The inclusion of the latter terms would only influence the numerical values of the couplings constants which we do not intend to calculate here.

We observe relations (67), neglect (72) and (68) in (65) and substitute (69) and (70) into the remaining terms of (65). Then with $\xi^n_\alpha X^t_{\alpha\beta}\xi^m_\beta \equiv X^t_{nm}$ after substitution of (65) into (63) one obtains

$$\begin{aligned}\mathcal{H}^1_{bf} = &-K_1\int \mathrm{d}^3z(\gamma^0\gamma^k)_{nm}(T^0\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^A_{k0}(\mathbf{z})\partial^f_{mj}(\mathbf{z})\\ &- K_1\int \mathrm{d}^3z(\gamma^0\gamma^k\gamma^5)_{nm}(S^0\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^G_{k0}(\mathbf{z})\partial^f_{mj}(\mathbf{z})\\ &+\frac{1}{3}K_1\sum_{b=1}^{3}\int \mathrm{d}^3z(\gamma^0\gamma^k)_{nm}(T^b\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^A_{kb}(\mathbf{z})\partial^f_{mj}(\mathbf{z})\\ &+\frac{1}{3}K_1\sum_{b=1}^{3}\int \mathrm{d}^3z(\gamma^0\gamma^k\gamma^5)_{nm}(S^b\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^G_{kb}(\mathbf{z})\partial^f_{mj}(\mathbf{z}),\end{aligned} \tag{73}$$

where the constant K_1 symbolizes the scalar products of the orbit states which can be defined by means of (70).

7. Coupling of Effective Currents to Bosons

The term responsible for the description of the interactions of effective currents with bosons is the second part of $\mathcal{H}_{bf}$ in (23). This part reads

$$\mathcal{H}^2_{bf} = W_4^{q_1q_2p_1p_2}R^k_{q_1q_2}b_k\partial^f_{p_1}\partial^f_{p_2} \tag{74}$$

with

$$\begin{aligned}W_4^{q_1q_2p_1p_2} := &-54W_{I_1I_2I_3K}F_{KI_4}R^{q_1}_{II'I_1}\\ &\cdot R^{q_2}_{K_1K_2I_4}C^{p_1}_{I'K_1K_2}C^{p_2}_{I_2I_3I}.\end{aligned} \tag{75}$$

Undoubtedly this term is the most complicated one with respect to the calculational effort for its evaluation.

To limit this calculational effort we use the s-wave approximation for the propagator from the outset. The latter can be read off from (47) and is given by

$$F^{\kappa\kappa'}_{\alpha\alpha'}(\mathbf{r})^s_{ii'} := \lambda_i\delta_{ii'}\gamma^5_{\kappa\kappa'}C_{\alpha\alpha'}g_{i'}(\mathbf{r}). \tag{76}$$

Then one obtains for (75) the formula

$$\begin{aligned}W_4^{q_1q_2p_1p_2} = &\int \mathrm{d}^3r_1\,\mathrm{d}^3r_2\,\mathrm{d}^3r_3\,\mathrm{d}^3r_4\,\mathrm{d}^3r\,\mathrm{d}^3r'\,\mathrm{d}^3z_1\,\mathrm{d}^3z_2\,\mathrm{d}^3z_3\,\lambda_{i_1}B_{i_2i_3i'_4}\lambda_{i'_4}\delta_{i'_4i_4}\\ &\cdot\sum_h\big[-(\gamma^0v^h)_{\beta_1\beta_2}(v^h)_{\beta_3\beta_4}\delta_{\varrho_1\varrho_2}\delta_{\varrho_3\varrho_4}+(\gamma^0v^h)_{\beta_1\beta_3}(v^h)_{\beta_2\beta_4}\delta_{\varrho_1\varrho_3}\delta_{\varrho_2\varrho_4}-(\gamma^0v^hC)_{\beta_1\beta_4}(v^hC)_{\beta_3\beta_2}\\ &\cdot\gamma^5_{\varrho_1\varrho_4}\gamma^5_{\varrho_3\varrho_2}\big]\delta(\mathbf{r}_1-\mathbf{r}_2)\delta(\mathbf{r}_1-\mathbf{r}_3)\delta(\mathbf{r}_1-\mathbf{r}_4)g_{i_4}(\mathbf{r}_4-\mathbf{z})R^{\varrho\varrho'\varrho_1}_{\beta\beta'\beta_1}(\mathbf{r},\mathbf{r}',\mathbf{r}_1|\mathbf{q}_1a_1\nu_1)_{ii'i_1}\\ &\cdot C^{\varrho_2\varrho_3\varrho}_{\beta_2\beta_3\beta}(\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}|\mathbf{p}_2b_2\mu_2)_{i_2i_3i}R^{\kappa_1\kappa_2\varrho_4}_{\alpha_1\alpha_2\beta_4}(\mathbf{z}_1,\mathbf{z}_2,\mathbf{z}|\mathbf{q}_2a_2\nu_2)_{i'_1i'_2i_4}C^{\kappa_1\kappa_2\varrho'}_{\alpha_1\alpha_2\beta'}(\mathbf{r}',\mathbf{z}_1\mathbf{z}_2|\mathbf{p}_1b_1\mu_1)_{i'_1i'_2i'},\end{aligned} \tag{77}$$

where the algebraic part of (76) has already been incorporated into the vertex expression.

For the free propagator the orbital part $g_i(\mathbf{r})$ in (76) is singular in $\mathbf{r}$ for single i, but the summation over i_4 in (77) corresponds to an intrinsic regularization of the propagator and the $\mathbf{z}$-part of the associated dual function in (77) and produces finite results.

The same must hold for the summation over the remaining auxiliary field indices $i_1, i_2, i_3, i, i', i_1', i_2'$, but details of this regularization process will be studied elsewhere. In this paper we rely on the less complicated examples, where it has been explicitly demonstrated that this regularization works well. Thus we omit the auxiliary field indices in the following, assuming that they guarantee finite values of the orbital scalar products.

The functions $g_i(\mathbf{r})$ as well as their regularized pendant $\hat{g}(\mathbf{r})$ are strongly localized around the origin $\mathbf{r} = 0$. As the integration of $\hat{g}(\mathbf{r})$ over $\mathbf{r}$ gives a finite value, one can expand the dual $R(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z})$ at the origin $\mathbf{z} = \mathbf{r}_1$ of $\hat{g}(\mathbf{r}_1 - \mathbf{z})$ and eliminate $\hat{g}$ by integration.

In addition in (77) the γ^5-terms drop out if applied to the superspin-isospin part of the leptonic wave functions, and the remaining two terms of the vertex can be reduced to the first term only, by interchange of the ϱ- and the β-indices in the second term. By means of these operations (77) is reduced to the expression:

$$
\begin{aligned}
\mathcal{H}^2_{bf} = {} & -2 \int \mathrm{d}^3 r_1 \mathrm{d}^3 r \mathrm{d}^3 r' \mathrm{d}^3 z_1 \mathrm{d}^3 z_2 \mathrm{d}^3 q_1 \mathrm{d}^3 q_2 \mathrm{d}^3 p_1 \mathrm{d}^3 p_2 \mathrm{d}^3 k \Big\{ \sum_h (\gamma^0 v^h)_{\beta_1\beta_2} (v^h)_{\beta_3\beta_4} \delta_{\varrho_1\varrho_2} \delta_{\varrho_3\varrho_4} \Big\} \\
& \cdot \exp[\mathrm{i}\mathbf{q}_1 \frac{1}{3}(\mathbf{r} + \mathbf{r}' + \mathbf{r}_1)] r^{\varrho\varrho'\varrho_1}_{\beta\beta'\beta_1}(\mathbf{r}, \mathbf{r}', \mathbf{r}_1 | a_1\nu_1) \exp[-\mathrm{i}\mathbf{p}_2 \frac{1}{3}(\mathbf{r}_1 + \mathbf{r}_1 + \mathbf{r})] \\
& \cdot c^{\varrho_2\varrho_3\varrho}_{\beta_2\beta_3\beta}(\mathbf{r}_1, \mathbf{r}_1, \mathbf{r} | b_2\mu_2) \exp[\mathrm{i}\mathbf{q}_2 \frac{1}{3}(\mathbf{z}_1 + \mathbf{z}_2 + \mathbf{r}_1)] r^{\kappa_1\kappa_2\varrho_4}_{\alpha_1\alpha_2\beta_4}(\mathbf{z}_1, \mathbf{z}_2, \mathbf{r}_1 | a_2\nu_2) \exp[-\mathrm{i}\mathbf{p}_1 \frac{1}{3}(\mathbf{z}_1 + \mathbf{z}_2 + \mathbf{r}')] \\
& \cdot c^{\kappa_1\kappa_2\varrho'}_{\alpha_1\alpha_2\beta'}(\mathbf{r}', \mathbf{z}_1, \mathbf{z}_2 | b_1\mu_1) \delta(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{k}) \psi^{a_1 a_2}_{\nu_1\nu_2}(\mathbf{q}_2 - \mathbf{q}_1 | \mathbf{k}, k) b(\mathbf{k}|k) \partial^f(\mathbf{p}_1 | b_1\mu_1) \partial^f(\mathbf{p}_2 | b_2\mu_2),
\end{aligned}
\tag{78}
$$

where in the s-wave approximation the internal wave functions do not depend on the center of mass momentum, and where

$$
R(\mathbf{q}_1, \mathbf{q}_2 | \mathbf{k}, k) := \delta(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{k}) \psi^{a_1 a_2}_{\nu_1\nu_2}(\mathbf{q}_2 - \mathbf{q}_1 | \mathbf{k}, k) \tag{79}
$$

are the dual boson functions in momentum space with general quantum number k.

In the next step in (78) the products of the internal fermion wave functions and of their duals are to be analyzed.

These wave functions are defined by (35) – (38) together with the spin part (39). If these representations are substituted in (78) the resulting products can be simplified by means of (72). With (39) the latter relation can be more precisely expressed in the form

$$
\lim_{\mathbf{r}', \mathbf{r}'' \to \mathbf{r}} C_{22} \{ C_{\beta\beta'} \xi^z_{\beta''} \psi(\mathbf{r}, \mathbf{r}', \mathbf{r}'') \} \equiv 0. \tag{80}
$$

Thus we can apply the arguments of Section 6 to neglect in (78) all C_{22}-terms in comparison with the leading C_{21}-terms. This leads to

$$
\begin{aligned}
& r^{\varrho\varrho_1\varrho'}_{\beta\beta_1\beta'}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}' | a_1\nu_1) (\gamma^0 v^h)_{\beta_1\beta_2} c^{\varrho\varrho_1\varrho_3}_{\beta\beta_2\beta_3}(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_1 | b_2\mu_2) (v^h)_{\beta_3\beta_4} r^{\kappa_1\kappa_2\varrho_3}_{\alpha_1\alpha_2\beta_4}(\mathbf{z}_1, \mathbf{z}_2, \mathbf{r}_1 | a_2\nu_2) c^{\kappa_1\kappa_2\varrho'}_{\alpha_1\alpha_2\beta'}(\mathbf{r}', \mathbf{z}_1, \mathbf{z}_2 | b_1\mu_1) \\
& = [(C_{21} \Theta^{a_1}_{\varrho\varrho_1\varrho'}) C_{12} \{ \bar{\Omega}^{\nu_1}_{\beta\beta_1\beta'} \psi^*(\mathbf{r}, \mathbf{r}_1, \mathbf{r}') \}] (\gamma^0 v^h)_{\beta_1\beta_2} [(C_{21} \Theta^{b_2}_{\varrho\varrho_1\varrho_3}) C_{12} \{ \Omega^{\mu_2}_{\beta\beta_2\beta_3} \psi(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_1) \}] (v^h)_{\beta_3\beta_4} \\
& \quad \cdot [(C_{21} \Theta^{a_2}_{\kappa_1\kappa_2\varrho_3}) C_{12} \{ \bar{\Omega}^{\nu_2}_{\alpha_1\alpha_2\beta_4} \psi^*(\mathbf{z}_1, \mathbf{z}_2, \mathbf{r}_1) \}] [(C_{21} \Theta^{b_1}_{\kappa_1\kappa_2\varrho'}) C_{12} \{ \Omega^{\mu_1}_{\alpha_1\alpha_2\beta'} \psi(\mathbf{r}', \mathbf{z}_1, \mathbf{z}_2) \}].
\end{aligned}
\tag{81}
$$

Formula (81) shows that the superspin-isospin part can be separately treated from the spin-orbit part. Its detailed evaluation depends on the kind of fermion functions appearing in (81). In particular the two dual r-functions can be considered as the input of a calculation which fixes the two c-functions as the output. This input is not arbitrary: To get a nonvanishing contribution to the boson sector of the effective theory according to (74), the set of two r-functions has to constitute the basis for a higher order representation of the electroweak boson wave functions.

To secure the relation of the higher order state representations to the hard core boson states, it is necessary that the three-fermion basis states possess nonvanishing projections on the one-fermion states which constitute the boson functions (27), (32). Otherwise they would be no elements of the polarization cloud.

Accidentally quark states as three-fermion (parton) states have the same global algebraic quantum numbers as one-parton states. Thus to distinguish quark states from one-parton states, the quark states are subjected to additional boundary conditions. These conditions prevent quark states from being projected into the one-parton sector and guarantee their linear independence from this sector [15]. If these boundary conditions are lifted the resulting "pseudo quark" three-fermion states are suitable canditates for the representation of higher order boson states.

For convenience we perform this calculation with D-conjugated spinors instead of S-conjugated ones, as the group theoretical representations of the three-parton states in D-conjugated spinors are more transparent than that where S-conjugated spinors are used.

According to (27) for bosons too the superspin-isospin part can be separated from the spin-orbit part. Thus the calculation of the whole superspin-isospin part of (78) or (81), respectively, can be completely decoupled from all other calculations. In particular with respect to the D-transformation the following identity holds

$$\begin{aligned}&(C_{21}\Theta^{a_1}_{\varrho\varrho_1\varrho'})^S(C_{21}\Theta^{b_2}_{\varrho\varrho_1\varrho_3})^S(C_{21}\Theta^{a_2}_{\kappa_1\kappa_2\varrho_3})^S\\ &\cdot(C_{21}\Theta^{b_1}_{\kappa_1\kappa_2\varrho'})^S\equiv(C_{21}\Theta^{a_1}_{\varrho\varrho_1\varrho'})^{\mathrm{D}}(C_{21}\Theta^{b_2}_{\varrho\varrho_1\varrho_3})^{\mathrm{D}}\\ &\cdot(C_{21}\Theta^{a_2}_{\kappa_1\kappa_2\varrho_3})^{\mathrm{D}}(C_{21}\Theta^{b_1}_{\kappa_1\kappa_2\varrho'})^{\mathrm{D}}\end{aligned}\tag{82}$$

as the dual functions transform reciprocally to their basis functions. In (82) the dual input functions are defined to be $(C_{21}\Theta^{a_1})$ and $(C_{21}\Theta^{a_2})$. They are given in D-representation by

$$\begin{aligned}(C_{21}\Theta^5)^{\mathrm{D}}&:=\chi^{1/2}_{1/2}(r_1)\otimes(1,1,2),\\ (C_{21}\Theta^6)^{\mathrm{D}}&:=\chi^{1/2}_{-1/2}(r_1)\otimes(1,1,2),\\ (C_{21}\Theta^7)^{\mathrm{D}}&:=\chi^{1/2}_{1/2}(r_1)\otimes(1,2,2),\\ (C_{21}\Theta^8)^{\mathrm{D}}&:=\chi^{1/2}_{-1/2}(r_1)\otimes(1,2,2),\end{aligned}\tag{83}$$

with

$$\begin{aligned}(1,1,2):=\left(\frac{1}{3}\right)^{1/2}&[\delta_{1\Lambda_1}\delta_{1\Lambda_2}\delta_{2\Lambda_3}\\ &+\delta_{1\Lambda_1}\delta_{2\Lambda_2}\delta_{1\Lambda_3}+\delta_{2\Lambda_1}\delta_{1\Lambda_2}\delta_{1\Lambda_3}],\\ (1,2,2):=\left(\frac{1}{3}\right)^{1/2}&[\delta_{2\Lambda_1}\delta_{2\Lambda_2}\delta_{1\Lambda_3}\\ &+\delta_{1\Lambda_1}\delta_{2\Lambda_2}\delta_{2\Lambda_3}+\delta_{2\Lambda_1}\delta_{1\Lambda_2}\delta_{2\Lambda_3}],\end{aligned}\tag{84}$$

and have the same quantum numbers as one-parton states (see [24, 25]).

To perform an exact evaluation of (82) the state numbers of the lepton states (37) and the pseudoquark states (83) have to be specified in accordance with the group theoretical representations in (37) and (83). Owing to the product states in (37) and (83) we introduce two quantum numbers $\hat{A}$ and $\hat{a}$ for the state characterization:

$$\begin{aligned}&\hat{A}=1\equiv(1,1,1),\quad \hat{A}=2\equiv(2,2,2),\\ &\hat{A}=3\equiv(1,1,2),\quad \hat{A}=4\equiv(1,2,2),\\ &\hat{a}=1\equiv\chi^{1/2}_{1/2}(r_1),\quad \hat{a}=2\equiv\chi^{1/2}_{-1/2}(r_1).\end{aligned}\tag{85}$$

In the following the hat-symbols of A and a will be omitted as from the use of these letters in the subsequent calculations a mistake is excluded with their use in Section 2 and (38). Furthermore the superspin-isospin indices κ,ϱ in the preceding formulas (78), (79), etc. with four components are now used as mere isospin indices with two components. Then with double indexing one obtains for the right hand side of (82)

$$\begin{aligned}&(C_{21}\Theta^{A_1a_1}_{l_1\lambda_1,l_2\lambda_2,r'\varrho'})^{\mathrm{D}}(C_{21}\Theta^{B_2b_2}_{l_1\lambda_1,l_2\lambda_2,r_3\varrho_3})^{\mathrm{D}}(C_{21}\Theta^{A_2a_2}_{k_1\kappa_1,k_2\kappa_2,r_3\varrho_3})^{\mathrm{D}}(C_{21}\Theta^{B_1b_1}_{k_1\kappa_1,k_2\kappa_2,r'\varrho'})^{\mathrm{D}}=\\ &s(l_1,l_2,r')^{A_1}s(l_1,l_2,r_3)^{B_2}s(k_1,k_2,r_3)^{A_2}s(k_1,k_2,r')^{B_1}\\ &\cdot(C_{21}\chi^{a_1}_{\lambda_1\lambda_2\varrho'})(C_{21}\chi^{b_2}_{\lambda_1\lambda_2\varrho_3})(C_{21}\chi^{a_2}_{\kappa_1\kappa_2\varrho_3})(C_{21}\chi^{b_1}_{\kappa_1\kappa_2\varrho'})=:S^{A_1a_1,A_2a_2}_{B_1b_1,B_2b_2},\end{aligned}\tag{86}$$

where the s-terms mean the symmetric superspin tensors in (38) and (84), and where the χ-terms are to be identified with the isospin tensors in (38).

Equation (86) is the starting point for the exact evaluation of this expression. With $[A]:=A$ modulo 2, one obtains

$$S^{A_1a_1,A_2a_2}_{B_1b_1,B_2b_2}=\left(\frac{5}{27}\right)[\delta_{3A_1}\delta_{4A_2}+\delta_{4A_1}\delta_{3A_2}]\delta_{[A_1]B_2}\delta_{[A_2]B_1}[\sigma^0_{a_1a_2}+\sigma^1_{a_1a_2}]\delta_{a_1b_1}\delta_{a_2b_2}.\tag{87}$$

In the last step of the calculation of the superspin-isospin part, we apply the corresponding boson tensors to (87) or (86), respectively. For charge conjugated spinors the superspin-isospin tensors of the bosons are given by (25) and (26). As the calculation of (87) is referred to G-conjugated spinors, it is necessary to transform the boson states to G-conjugated spinors too.

In accordance with (37) and (38) in this representation double indexing has to be introduced and as the boson states are coupled to the fermion state numbers (85) the notation has to be adapted to (85) by replacing (Λ, A) of (38) by $(\hat{A}, a)$.

Omitting the hat on A, the superspin-isospin part of the boson states results from the combination of the symmetric or antisymmetric superspin states, respectively, for $f = 0$. They are given by

$$\begin{aligned} s_+ &:= (\delta_{1A_1}\delta_{2A_2} + \delta_{2A_1}\delta_{1A_2}), \\ s_- &:= (\delta_{1A_1}\delta_{2A_2} - \delta_{2A_1}\delta_{1A_2}) \end{aligned} \tag{88}$$

with the isospin singlet

$$\Psi_0^0 := (\delta_{1a_1}\delta_{2a_2} - \delta_{2a_1}\delta_{1a_2}) \tag{89}$$

or the isospin triplet

$$\begin{aligned} \Psi_0^1 &:= (\delta_{1a_1}\delta_{2a_2} + \delta_{2a_1}\delta_{1a_2}), \\ \Psi_1^1 &:= \delta_{1a_1}\delta_{1a_2}, \quad \Psi_{-1}^1 := \delta_{2a_1}\delta_{2a_2}, \end{aligned} \tag{90}$$

where the combined superspin-isospin states are direct products of (88), (89) or (88), (90), respectively.

One gets for the linear combination of corresponding symmetric and antisymmetric states

$$\begin{aligned} (\Theta^0)^{\mathrm{D}} &:= \Theta_0^0(a) + \Theta_0^0(s) = 2\Psi_0^0\, \delta_{1A_1}\delta_{2A_2}, \\ (\Theta^3)^{\mathrm{D}} &:= \Theta_0^1(a) + \Theta_0^1(s) = 2\Psi_0^1\, \delta_{1A_1}\delta_{2A_2}, \\ (\Theta^1)^{\mathrm{D}} &:= \Theta_1^1(a) + \Theta_1^1(s) = 2\Psi_1^1\, \delta_{1A_1}\delta_{2A_2}, \\ (\Theta^2)^{\mathrm{D}} &:= \Theta_{-1}^1(a) + \Theta_{-1}^1(s) = 2\Psi_{-1}^1\, \delta_{1A_1}\delta_{2A_2}, \end{aligned} \tag{91}$$

where the numbers 0 and 3 denote electrically neutral states while 1 and 2 denote charged states. Apart from linear combinations the set (91) is equivalent to the set $(S^l + T^l)^{\mathrm{S}}$ and by construction of the effective theory the boson states are coupled to the fermion quantum numbers in (74).

The coupling of (91) to fermion numbers $A_1 = 3$, $A_2 = 4$ or $A_1 = 4$, $A_2 = 3$ in (87) requires an additional change of the index notation in the boson functions: one has to replace A_1, A_2 by $[A_1]$, $[A_2]$ which results directly from the calculation. Then one obtains with (82)–(84)

$$\begin{aligned} &\sum_{A_1A_2}\sum_{a_1a_2}(C_{21}\Theta^{A_1a_1}_{\varrho\varrho_1\varrho'})^{\mathrm{D}}(C_{21}\Theta^{B_2b_2}_{\varrho\varrho_1\varrho_3})^{\mathrm{D}}(C_{21}\Theta^{A_2a_2}_{\kappa_1\kappa_2\varrho_3})^{\mathrm{D}} \\ &\quad\cdot (C_{21}\Theta^{B_1b_1}_{\kappa_1\kappa_2\varrho'})^{\mathrm{D}}(\Theta^n_{[A_1]a_1,[A_2]a_2})^{\mathrm{D}} \\ &= \frac{10}{27}(\Theta^n_{B_1b_1,B_2b_2})^{\mathrm{D}}, \end{aligned} \tag{92}$$

where the summation on the left-hand side of (92) runs over those fermion state configurations which are admitted for the boson representation.

In a further step the spin-orbit coupling of the fermion states and their projection on the spin-orbit part of the boson states have to be analyzed. In so doing use is made of equation (70). If definition (39) is observed, combination of (70) with the spin-orbit part of (81) yields

$$\begin{aligned} &\sum_h C_{12}\{\bar{\Omega}^{\nu_1}_{\beta\beta_1\beta'}\psi^*(\mathbf{r},\mathbf{r}_1,\mathbf{r}')\}(\gamma^0 v^h)_{\beta_1\beta_2} C_{12}\{\Omega^{\mu_2}_{\beta\beta_2\beta_3}\psi(\mathbf{r},\mathbf{r}_1,\mathbf{r}_1)\}(v^h)_{\beta_3\beta_4} \\ &\quad\cdot C_{12}\{\bar{\Omega}^{\nu_2}_{\alpha_1\alpha_2\beta_4}\psi^*(\mathbf{z}_1,\mathbf{z}_2,\mathbf{r}_1)\} C_{12}\{\Omega^{\mu_1}_{\alpha_1\alpha_2\beta'}\psi(\mathbf{r}',\mathbf{z}_1,\mathbf{z}_2)\} \\ &= \sum_h C_{12}\{\bar{\Omega}^{\nu_1}_{\beta\beta_1\mu_1}\psi^*(\mathbf{r},\mathbf{r}_1,\mathbf{r}')\}(\gamma^0 v^h)_{\beta_1\beta_2} C_{12}\{\Omega^{\mu_2}_{\beta\beta_2\beta_3}\psi(\mathbf{r},\mathbf{r}_1,\mathbf{r}_1)\}(v^h)_{\beta_3\nu_2}\Phi \end{aligned} \tag{93}$$

with

$$\begin{aligned} \Phi := \{&\psi^*(\mathbf{r}_1-\mathbf{z}_1,\mathbf{z}_2-\mathbf{r}_1)[\psi(\mathbf{z}_1-\mathbf{z}_2,\mathbf{r}'-\mathbf{z}_1)+\psi(\mathbf{z}_2-\mathbf{z}_1,\mathbf{r}'-\mathbf{z}_2)] \\ &+\psi^*(\mathbf{z}_2-\mathbf{r}_1,\mathbf{z}_1-\mathbf{z}_2)[\psi(\mathbf{z}_2-\mathbf{r}',\mathbf{z}_1-\mathbf{z}_2)+\psi(\mathbf{r}'-\mathbf{z}_2,\mathbf{z}_1-\mathbf{r}')] \\ &+\psi^*(\mathbf{r}_1-\mathbf{z}_2,\mathbf{z}_1-\mathbf{r}_1)[\psi(\mathbf{z}_2-\mathbf{r}',\mathbf{z}_1-\mathbf{z}_2)+\psi(\mathbf{r}'-\mathbf{z}_2,\mathbf{z}_1-\mathbf{r}')] \\ &+\psi^*(\mathbf{z}_1-\mathbf{r}_1,\mathbf{z}_2-\mathbf{z}_1)[\psi(\mathbf{z}_1-\mathbf{z}_2,\mathbf{r}'-\mathbf{z}_1)+\psi(\mathbf{z}_2-\mathbf{z}_1,\mathbf{r}'-\mathbf{z}_2)]\}. \end{aligned} \tag{94}$$

The additional second set of terms is in this case nontrivial and will be discussed at the end of this section.

Expression (93) can be exactly evaluated. For the sake of brevity we suppress details of this calculation and consider the results after the boson functions have been projected on (93).

With respect to the spin-orbit part this means that the boson states must be coupled to the fermion momenta which enforces the use of the momentum space representations of the boson wave functions (33) referred to G-conjugated spinors; see (78). With

$$\begin{aligned} I^{\nu_1\mu_1\nu_2\mu_2} &:= \sum_h C_{12}\{\bar{\Omega}^{\nu_1}_{\beta\beta_1\mu_1}\psi^*(\mathbf{r},\mathbf{r}_1,\mathbf{r}')\}(\gamma^0 v^h)_{\beta_1\beta_2}\{\Omega^{\mu_2}_{\beta\beta_2\beta_3}\psi(\mathbf{r},\mathbf{r}_1,\mathbf{r}_1)\}(v^h)_{\beta_3\nu_2} \\ &= \{[\delta_{\nu_2\mu_1}\gamma^0_{\nu_1\mu_2} - \gamma^5_{\nu_2\mu_1}(\gamma^0\gamma^5)_{\nu_1\mu_2}]\Phi\Pi_1 + [C_{\nu_1\nu_2}(\gamma^0 C)_{\mu_1\mu_2} - (\gamma^5 C)_{\nu_1\nu_2}(\gamma^0\gamma^5 C)_{\mu_1\mu_2}]\Phi\Pi_2\} \end{aligned} \tag{95}$$

one obtains for the parts of the vector potentials of (33) in temporal gauge

$$I^{\nu_1\mu_1\nu_2\mu_2}\tilde{f}^A(\mathbf{q}_2-\mathbf{q}_1)(\gamma^k C)^+_{\nu_1\nu_2} \equiv 0, \quad k=1,2,3 \tag{96}$$

and

$$I^{\nu_1\mu_1\nu_2\mu_2}\tilde{f}^G(\mathbf{q}_2-\mathbf{q}_1)(\gamma^5\gamma^k C)^+_{\nu_1\nu_2} \equiv 0, \quad k=1,2,3. \tag{97}$$

With respect to the field strength parts of (33) the decomposition (43) into electric and magnetic field parts is used which leads to the nonvanishing relations

$$-\mathrm{i}I^{\nu_1\mu_1\nu_2\mu_2}\tilde{f}^E(\mathbf{q}_2-\mathbf{q}_1)(\gamma^0\gamma^k C)^+_{\nu_1\nu_2} = 2\mathrm{i}\tilde{f}^E(\mathbf{q}_2-\mathbf{q}_1)(\gamma^k C)^+_{\mu_1\mu_2}\Pi_1, \quad k=1,2,3 \tag{98}$$

and

$$-\mathrm{i}I^{\nu_1\mu_1\nu_2\mu_2}\tilde{f}^B(\mathbf{q}_2-\mathbf{q}_1)(\gamma^l\gamma^j C)^+_{\nu_1\nu_2} = -2\mathrm{i}\tilde{f}^B(\mathbf{q}_2-\mathbf{q}_1)i\varepsilon_{ljk}(\gamma^5\gamma^k C)^+_{\mu_1\mu_2}\Pi_1 \tag{99}$$

for $l<j, l,j=1,2,3$, and where Π_1 is defined by

$$\Pi_1 := [\psi^*(\mathbf{r}'-\mathbf{r},\mathbf{r}_1-\mathbf{r}') + \psi^*(\mathbf{r}-\mathbf{r}',\mathbf{r}_1-\mathbf{r})][\psi(\mathbf{r}_1-\mathbf{r},0) + \psi(\mathbf{r}-\mathbf{r}_1,\mathbf{r}_1-\mathbf{r})]. \tag{100}$$

The part with Π_2 in (95) vanishes for all boson projectors owing to orthogonality relations. In this way all algebraic constituents of (78) have been exactly calculated.

In the last step the above results are substituted in (78) and the remaining integrals have to be calculated approximately. Substitution of (92) and (98), (99) into (78) yields

$$\begin{aligned} \mathcal{H}^2_{bf} = \int & \mathrm{d}^3r_1\,\mathrm{d}^3r\,\mathrm{d}^3r'\,\mathrm{d}^3z_1\,\mathrm{d}^3z_2\,\mathrm{d}^3q_1\,\mathrm{d}^3q_2\,\mathrm{d}^3p_1\,\mathrm{d}^3p_2\,\mathrm{d}^3k\,[\Phi\Pi_1]\exp[\mathrm{i}\mathbf{q}_1\frac{1}{3}(\mathbf{r}+\mathbf{r}'+\mathbf{r}_1)] \\ & \cdot\ \exp[-\mathrm{i}\mathbf{p}_2\frac{1}{3}(2\mathbf{r}_1+\mathbf{r})]\exp[\mathrm{i}\mathbf{q}_2\frac{1}{3}(\mathbf{z}_1+\mathbf{z}_2+\mathbf{r}_1)]\exp[-\mathrm{i}\mathbf{p}_1\frac{1}{3}(\mathbf{z}_1+\mathbf{z}_2+\mathbf{r}')]\delta(\mathbf{q}_1+\mathbf{q}_2-\mathbf{k}) \\ & \cdot\ \Theta^n_{B_1b_1,B_2b_2}\mathrm{i}[2\tilde{f}^E(\mathbf{q}_1-\mathbf{q}_2)(\gamma^k C)^+_{\mu_1\mu_2}b^E(\mathbf{k}|n,k) - 2\tilde{f}^B(\mathbf{q}_2-\mathbf{q}_1)\mathrm{i}\varepsilon_{ljk}(\gamma^5\gamma^k C)^+_{\mu_1\mu_2} \\ & \cdot\ b^B(\mathbf{k}|n,l,j)]\partial^f(\mathbf{p}_1|B_1,b_1,\mu_1)\partial^f(\mathbf{p}_2|B_2,b_2,\mu_2). \end{aligned} \tag{101}$$

To get an idea about the consequences of the integrations in (101) with the kernel $[\Phi\Pi]$, we simulate the fermionic groundstates by the direct product of test functions $t(\mathbf{r})$ in the internal coordinates. The latter functions are assumed to be strongly concentrated around the origin (approximate δ-distributions), but with finite values $t(0)$ at the origin which reflect the intrinsic regularization of the true groundstate functions involved. In this way the groundstate function is defined by

$$\psi(\mathbf{u},\mathbf{v}) := t(\mathbf{u})t(\mathbf{v}). \tag{102}$$

Then the integrations over $\mathbf{r}_1, \mathbf{r}', \mathbf{r}, \mathbf{z}_1, \mathbf{z}_2$ can be performed and give for (101) the expression

$$(78) = K \int \mathrm{d}^3 z_1\, \mathrm{d}^3 q_1\, \mathrm{d}^3 q_2\, \mathrm{d}^3 p_1\, \mathrm{d}^3 p_2\, \mathrm{d}^3 k\, t(0)^4 \exp[\mathrm{i}(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{p}_1 - \mathbf{p}_2)\mathbf{z}_1]\delta(\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{k}) \cdot \Theta^n_{B_1 b_1, B_2 b_2} \mathrm{i}[2\tilde{f}^E(\mathbf{q}_2 - \mathbf{q}_1)(\gamma^k C)^+_{\mu_1\mu_2} b^E(\mathbf{k}|n,k) - 2\tilde{f}^B(\mathbf{q}_2 - \mathbf{q}_1)\mathrm{i}\varepsilon_{ljk}(\gamma^5\gamma^k C)^+_{\mu_1\mu_2} \cdot b^B(\mathbf{k}|n,l,j)]\partial^f(\mathbf{p}_1|B_1, b_1, \mu_1)\partial^f(\mathbf{p}_2|B_2, b_2, \mu_2). \quad (103)$$

Introducing the new coordinates $\mathbf{w} = \mathbf{q}_1 + \mathbf{q}_2$ and $\mathbf{u} = \mathbf{q}_2 - \mathbf{q}_1$, after integration over $\mathbf{w}$ and $\mathbf{u}$ and using the definition of b^B_m in (43) one obtains the final formula

$$\mathcal{H}^2_{bf} = \mathrm{i}Kt(0)^4 \int \mathrm{d}^3 z \Theta^n_{B_1 b_1, B_2 b_2}[2f^E(\gamma^k C)^+_{\mu_1\mu_2} b^E(\mathbf{z}|n,k) + f^B i(\gamma^5\gamma^k C)^+_{\mu_1\mu_2} b^B(\mathbf{z}|n,k)]\partial^f(\mathbf{z}|B_1, b_1, \mu_1)\partial^f(\mathbf{z}|B_2, b_2, \mu_2), \quad (104)$$

where f^E and f^B result from the integration over the associated boson functions.

The second set of terms resulting from the left-hand side of (93) can be treated in an analogous way. In addition to terms like (104), coupling terms of the leptonic electric and magnetic dipole moments to the boson fields appear. Their counterparts in the coupling of bosons to these leptonic dipole moments in the effective fermion equations are not present in (73). But it can be shown that even these terms can be derived from (74) if mixed couplings are taken into account. A discussion of this extension of the formalism would exceed the scope of this paper and will be done elsewhere.

8. Summary and Physical Interpretation

In the notation of the preceding sections the leading terms of the functional energy operator (22) read

$$\tilde{\mathcal{H}} = \mathcal{H}_f + \mathcal{H}^1_b + \mathcal{H}^2_b + \mathcal{H}^3_b + \mathcal{H}^1_{bf} + \mathcal{H}^2_{bf}. \quad (105)$$

Except for $\mathcal{H}_f$ all other terms have been calculated and the results of these calculations are formulated by equations (45), (48), (55), (73) and (104). The term $\mathcal{H}_f$ can be easily derived by means of (65) – (70). It reads

$$\mathcal{H}_f = \int \mathrm{d}^3 z f(\mathbf{z}|B_1 b_1 \alpha_1)[-\mathrm{i}(\gamma^0\gamma^k)\partial^z_k + m\gamma^0]_{\alpha_1\alpha_2}\partial^f(\mathbf{z}|B_1 b_1 \alpha_2). \quad (106)$$

Substituting (105) and (106) with (45), (48), (55), (73) and (104) into (21) one obtains the effective functional energy equation which describes the physical behavior of the phenomenological system apart from quantization terms and higher order terms.

A physical interpretation of the effective functional energy equation can be achieved by considering the classical limit of this equation. In this classical limit the system is described by the associated classical equations of motion. These equations of motion can be exactly derived from (21) if correlations in the matrix elements are suppressed. The corresponding deduction has already been published repeatedly. So in the case under consideration we give only the result of this method and refer for details to [9], Section 7.5 for instance. With (53) and with $\boldsymbol{A}_a$:= electric vector potential, $\boldsymbol{G}_a$:= magnetic vector potential, $\boldsymbol{E}_a$:= electric field strength, $\boldsymbol{B}_a$:= magnetic field strength, the associated classical equations of motion of the vector fields read

$$\mathrm{i}\dot{A}_{la}(\mathbf{z}) = c_1\varepsilon_{klm}\partial^z_k G_{ma}(\mathbf{z}) + \mathrm{i}c_2 E_{la}(\mathbf{z}) - \eta_{abc}\varepsilon_{klm}[\hat{f}^A \mathrm{i}k_1 A_{kb}(\mathbf{z})G_{mc}(\mathbf{z}) + \hat{f}^G \mathrm{i}k_4 G_{kb}(\mathbf{z})A_{mc}(\mathbf{z})], \quad (107)$$

$$\mathrm{i}\dot{G}_{la}(\mathbf{z}) = c_1\varepsilon_{klm}\partial^z_k A_{ma}(\mathbf{z}) + c_3 B_{la}(\mathbf{z}) - \eta_{abc}\varepsilon_{klm}[\hat{f}^A \mathrm{i}k_3 A_{kb}(\mathbf{z})A_{mc}(\mathbf{z}) + \hat{f}^G \mathrm{i}k_6 G_{kb}(\mathbf{z})G_{ma}(\mathbf{z})], \quad (108)$$

$$\mathrm{i}\dot{E}_{la}(\mathbf{z}) = \mathrm{i}\varepsilon_{klm}\partial^z_k B_{ma}(\mathbf{z}) - \mathrm{i}(c_2 - \hat{f}^A c_4)A_{la}(\mathbf{z}) + \eta_{abc}\varepsilon_{klm}[\hat{f}^A k_2 A_{kb}(\mathbf{z})B_{mc}(\mathbf{z}) - \hat{f}^G \mathrm{i}k_5 G_{kb}(\mathbf{z})E_{mc}(\mathbf{z})] + \mathrm{i}K'\Theta^a_{B_1 b_1, B_2 b_2}\hat{f}^E(\gamma^l C)^+_{\mu_1\mu_2}\psi_{B_1 b_1 \mu_1}(\mathbf{z})\psi_{B_2 b_2 \mu_2}(\mathbf{z}), \quad (109)$$

$$\mathrm{i}\dot{B}_{la}(\mathbf{z}) = -\mathrm{i}\varepsilon_{klm}\partial_k^z E_{ma}(\mathbf{z}) + (c_3 - \hat{f}^G c_4)G_{la}(\mathbf{z}) + \eta_{abc}\varepsilon_{klm}[-\hat{f}^A k_2 A_{kb}(\mathbf{z})E_{mc}(\mathbf{z}) - \mathrm{i}\hat{f}^G k_5 G_{kb}(\mathbf{z})B_{mc}(\mathbf{z})] + \mathrm{i}(K'/2)\Theta^a_{B_1b_1,B_2b_2}\hat{f}^B(\mathrm{i}\gamma^5\gamma^l C)^+_{\mu_1\mu_2}\psi_{B_1b_1\mu_1}(\mathbf{z})\psi_{B_2b_2\mu_2}(\mathbf{z}). \tag{110}$$

For the fermion fields the following equations of motion can be derived:

$$\mathrm{i}\dot{\psi}_{\alpha l}(\mathbf{z}) = \big[-\mathrm{i}(\gamma^0\gamma^k)_{\alpha\beta}\partial_k^z + \gamma^0_{\alpha\beta}m\big]\psi_{\beta l}(\mathbf{z}) + K_1\big[(\gamma^0\gamma^k)_{\alpha\beta}(T^0\gamma^5)_{lj}A_{k0}(\mathbf{z}) + (\gamma^0\gamma^k\gamma^5)_{\alpha\beta}(S^0\gamma^5)_{lj}G_{k0}(\mathbf{z})\big]\psi_{\beta j}(\mathbf{z}) - \frac{1}{3}K_1\sum_{b=1}^{3}\big[(\gamma^0\gamma^k)_{\alpha\beta}(T^b\gamma^5)_{lj}A_{kb}(\mathbf{z}) + (\gamma^0\gamma^k\gamma^5)_{\alpha\beta}(S^b\gamma^5)_{lj}G_{kb}(\mathbf{z})\big]\psi_{\beta j}(\mathbf{z}), \tag{111}$$

where to simplify matters only the single superspin-isospin index has been applied.

Equations (107) – (111) contain a lot of constants which complicate the physical interpretation. To reduce the number of independent constants, additional constraints have to be introduced and evaluated.

In [10] it was demonstrated that electric and magnetic vector bosons simultaneously possess a vanishing rest mass before isospin symmetry breaking is performed. It is reasonable to impose this condition also on the effective theory as long as the vacuum state (17) is assumed to conserve isospin symmetry. Under this assumption the calculations in the preceding sections have been performed.

In the effective theory the mass term of the electric vector bosons is given by $(c_2 - \hat{f}^A c_4)$ in equation (109), while the mass term for magnetic vector bosons results from $i(c_3 - \hat{f}^G c_4)$ in equation (110). The values of the mass terms of these bosons depend on the internal structure of the bosons which determines their mass eigenvalue equation. Without further investigating these boson wave functions we assume that equal masses require equal internal wave functions at least in s-wave approximation, i.e., we assume $f^G \equiv f^A$ and thus $\hat{f}^G = \hat{f}^A$. This on the other hand requires $r^G \equiv r^A$. Then from equations (44) and (54) it can be concluded that $c_1 = 1$, $c_2 = c_3$ and $k_1 = k_6$, $k_3 = k_4$, $k_1 = k_4$ and $k_2 = k_5$ must hold.

If furthermore the electric and magnetic field strengths are renormalized by $c_2 E = E'$ and $c_2 B = B'$ and if by an appropriate choice of the coupling constant g the vanishing of the electric vector boson mass $(c_2 - \hat{f}^A c_4)$ is enforced, then the mass of the magnetic vector boson must vanish too, and under these premises equations (107) – (110) go over into the following set of equations:

$$\mathrm{i}\dot{A}_{la}(\mathbf{z}) = \varepsilon_{klm}\partial_k^z G_{ma}(\mathbf{z}) + \mathrm{i}E'_{la}(\mathbf{z}) - \mathrm{i}K''\eta_{abc}\varepsilon_{klm}[A_{kb}(\mathbf{z})G_{mc}(\mathbf{z}) + G_{kb}(\mathbf{z})A_{mc}(\mathbf{z})], \tag{112}$$

$$\mathrm{i}\dot{G}_{la}(\mathbf{z}) = \varepsilon_{klm}\partial_k^z A_{ma}(\mathbf{z}) + B'_{la}(\mathbf{z}) - \mathrm{i}K''\eta_{abc}\varepsilon_{klm}[A_{kb}(\mathbf{z})A_{mc}(\mathbf{z}) + G_{kb}(\mathbf{z})G_{ma}(\mathbf{z})], \tag{113}$$

$$\mathrm{i}\dot{E}'_{la}(\mathbf{z}) = \mathrm{i}\varepsilon_{klm}\partial_k^z B'_{ma}(\mathbf{z}) + K\eta_{abc}\varepsilon_{klm}[A_{kb}(\mathbf{z})B'_{mc}(\mathbf{z}) - \mathrm{i}G_{kb}(\mathbf{z})E'_{mc}(\mathbf{z})] + \mathrm{i}K'\Theta^a_{B_1b_1,B_2b_2}\hat{f}^E(\gamma^l C)^+_{\mu_1\mu_2}\psi_{B_1b_1\mu_1}(\mathbf{z})\psi_{B_2b_2\mu_2}(\mathbf{z}), \tag{114}$$

$$\mathrm{i}\dot{B}'_{la}(\mathbf{z}) = -\mathrm{i}\varepsilon_{klm}\partial_k^z E'_{ma}(\mathbf{z}) + K\eta_{abc}\varepsilon_{klm}[-A_{kb}(\mathbf{z})E'_{mc}(\mathbf{z}) - \mathrm{i}G_{kb}(\mathbf{z})B'_{mc}(\mathbf{z})] + \mathrm{i}(K'/2)\Theta^a_{B_1b_1,B_2b_2}\hat{f}^G(i\gamma^5\gamma^l C)^+_{\mu_1\mu_2}\psi_{B_1b_1\mu_1}(\mathbf{z})\psi_{B_2b_2\mu_2}(\mathbf{z}), \tag{115}$$

while equation (111) remains unchanged.

Equations (112) – (115) can be further simplified. After an additional symmetry breaking the W- and Z-bosons become massive, so that in low energy processes the contributions of the heavy vector bosons can be neglected in favor of the action of electric and magnetic photons. With respect to this symmetry breaking we refer to [9], chapter 8, and suppress in a first step the W-bosons, i. e., $\mathbf{A}_1$ and $\mathbf{A}_2$ in (112) – (115) without performing the symmetry breaking procedure explicitly.

The electric and magnetic photons arise by a mixture of the $\boldsymbol{A}_0$ and $\boldsymbol{A}_3$ and the $\boldsymbol{G}_0$ and $\boldsymbol{G}_3$ vector fields together with the Z-boson and a corresponding magnetic X-boson which both later on will be neglected too.

The omission of the $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ vector fields induces the vanishing of the terms with ε_{abc} in η_{abc}. This in turn, leads in (112) and (113) to the vanishing of the nonlinear terms, as ε_{klm} acts as an antisymmetric projector in k, m on a symmetric tensor in k, m. Thus from (112) and (113) one yields for $a = 0, 3$

$$\mathrm{i}\dot{\mathbf{A}}_a = \nabla \times \mathbf{G}_a + \mathrm{i}\mathbf{E}'_a, \quad \mathrm{i}\dot{\mathbf{G}}_a = \nabla \times \mathbf{A}_a + \mathbf{B}'_a. \tag{116}$$

On the other hand in equations (114) and (115) the field part cannot be further simplified, but the currents are to be rearranged in order to show its equivalence to the conventional expressions.

First one verifies the invariance relation

$$\begin{aligned}&\psi^G_{B_1 b_1 \mu_1} (\Theta^a_{B_1 b_1, B_2 b_2})^{\mathrm{D}} \psi^G_{B_2 b_2 \mu_2} \\ &= \psi^C_{B_1 b_1 \mu_1} (\Theta^a_{B_1 b_1, B_2 b_2})^S \psi^C_{B_2 b_2 \mu_2}\end{aligned} \tag{117}$$

holds. Then from (33) and (91) one obtains for $a = 0, 3$

$$\begin{aligned}(\Theta^0)^S &:= \frac{1}{2}(T^0 + S^0)^S = (\mathrm{i}\sigma^2 + \sigma^1) \otimes \sigma^0, \\ (\Theta^3)^S &:= \frac{1}{2}(T^3 + S^3)^S = (\mathrm{i}\sigma^2 + \sigma^1) \otimes \sigma^3\end{aligned} \tag{118}$$

and thus with (3) for $a = 0, 3$, the electric current

$$\begin{aligned}&\psi^G_{B_1 b_1 \mu_1} (\Theta^a_{B_1 b_1, B_2 b_2})^{\mathrm{D}} (\gamma^l C)^+_{\mu_1 \mu_2} \psi^G_{B_2 b_2 \mu_2} \\ &= \psi^C_{1 b_1 \mu_1} \sigma^a_{b_1 b_2} \gamma^l_{\mu_1 \mu_2} \psi^C_{2 b_2 \mu_2}.\end{aligned} \tag{119}$$

To interpret this formula we remember that according to the definition of quantum numbers in (85), ψ^C_{2b} corresponds to (e^-, ν), while ψ^C_{1b} corresponds to $(e^+, \bar{\nu})$. Therefore if we define the effective fields in accordance with the phenomenological fields of the Standard model by $\psi_{b\mu} := \psi^C_{2b\mu}$ and $\psi^c_{b\mu} := \psi^C_{1b\mu}$, then with $(\psi^c)^T = \bar{\psi} C^T$ the right hand side of equation (119) can be expressed in phenomenological fields as

$$\begin{aligned}&\psi^C_{1 b_1 \mu_1} \sigma^a_{b_1 b_2} \gamma^l_{\mu_1 \mu_2} \psi^C_{2 b_2 \mu_2} \\ &\equiv \bar{\psi}_{b_1 \mu_1} \sigma^a_{b_1 b_2} \gamma^l_{\mu_1 \mu_2} \psi_{b_2 \mu_2}.\end{aligned} \tag{120}$$

In an analogous way the magnetic current can be treated to get the corresponding phenomenological expression.

With these results equations (114) and (115) can equivalently be written as (omitting the primes)

$$\begin{aligned}\mathrm{i}\dot{\mathbf{E}}_a &= \mathrm{i}\nabla \times \mathbf{B}_a + \eta_{abc}[\mathbf{A}_b \times \mathbf{B}_c - \mathrm{i}\mathbf{G}_b \times \mathbf{E}_c] \\ &\quad + \mathrm{i}K\bar{\psi}\sigma^a\vec{\gamma}\psi,\end{aligned} \tag{121}$$

$$\begin{aligned}\mathrm{i}\dot{\mathbf{B}}_a &= -\mathrm{i}\nabla \times \mathbf{E}_a - \eta_{abc}[\mathbf{A}_b \times \mathbf{E}_c + \mathrm{i}\mathbf{G}_b \times \mathbf{B}_c] \\ &\quad + (\mathrm{i}K/2)\bar{\psi}\sigma^a(i\gamma^5\vec{\gamma})\psi.\end{aligned} \tag{122}$$

It can be easily verified that the linear parts of equations (116) and (121), (122) correspond to the theory of Cabibbo and Ferrari. As far as the nonlinear terms in (121) and (122) are concerned we compare them with the field equations of a nonabelian $SU(2)$-theory with unbroken symmetry which for the "electric" part are given by

$$\begin{aligned}\dot{\mathbf{E}}_a &= \nabla \times \mathbf{B}_a + g\varepsilon_{abc}(\mathbf{A}_b \times \mathbf{B}_c) - j^e_a, \\ \dot{\mathbf{B}}_a &= -\nabla \times \mathbf{E}_a - g\varepsilon_{abc}(\mathbf{A}_b \times \mathbf{E}_c), \\ \dot{\mathbf{A}}_a &= -\mathbf{E}_a.\end{aligned} \tag{123}$$

By comparison of these equations with (121) and (122) it can be concluded that the nonlinear terms in (121) and (122) correspond to relics of the broken $SU(2)$-symmetric interactions and their mixture with $U(1)$-parts owing to the form of η_{abc}.

Finally we rearrange the Dirac equation (111) into the conventional form. Equation (111) can be formulated in G-conjugated as well as in C-conjugated spinors as it is forminvariant under corresponding transformations. We choose the representation by C-conjugated spinors, define $(2/3)K_1 = g$, $2K_1 = g'$ and multiply (111) by γ^0 from the left. In addition in accordance with the treatment of the vector field part, we suppress the 1,2-vector fields in (111). Then (111) reads

$$\begin{aligned}&[-\mathrm{i}\gamma^\mu\partial_\mu + m]\psi^C \\ &+[-\frac{1}{2}g(\sigma^3 \otimes \sigma^3)\gamma^k A_{k3} + \frac{1}{2}g'(\sigma^3 \otimes \sigma^0)\gamma^k A_{k0}]\psi^C \\ &+[-\frac{1}{2}g(\sigma^0 \otimes \sigma^3)(\gamma^k\gamma^5)G_{k3} + \frac{1}{2}g'(\sigma^0 \otimes \sigma^0)G_{k0}]\psi^C \\ &\quad = 0.\end{aligned} \tag{124}$$

In particular for $\psi^C_{2a} \equiv \psi_a \equiv (e^-, \nu)$ one obtains from (124)

$$[-\mathrm{i}\gamma^\mu\partial_\mu + m]\psi_a + [\frac{1}{2}g\sigma^3_{ab}\gamma^k A_{k3} - \frac{1}{2}g'\sigma^0_{ab}\gamma^k A_{k0}]\psi_b$$

$$+ [-\frac{1}{2} g \sigma^3_{ab} (\gamma^k \gamma^5) G_{k3} + \frac{1}{2} g' \sigma^0_{ab} (\gamma^k \gamma^5) G_{k0}] \psi_b = 0. \quad (125)$$

To secure the consistency of the currents with the coupling constants g and g' in (125) we introduce the transformations $\psi' = K^{-1/2}\psi$ and

$$\mathbf{A}_3 = g^{-1/2}\mathbf{A}'_3, \qquad \mathbf{E}'_3 = g^{-1/2}\mathbf{E}''_3,$$
$$\mathbf{A}_0 = (g')^{-1/2}\mathbf{A}'_0, \quad \mathbf{E}'_0 = (g')^{-1/2}\mathbf{E}''_0, \quad (126)$$

and analogous relations for the magnetic fields.

If the transformations (126) and the Weinberg transformation

$$\mathbf{A}'_3 = \mathbf{Z}\cos\Theta - \mathbf{A}\sin\Theta, \ \mathbf{G}'_3 = \mathbf{X}\cos\Theta - \mathbf{G}\sin\Theta,$$
$$\mathbf{A}'_0 = \mathbf{Z}\sin\Theta + \mathbf{A}\cos\Theta, \ \mathbf{G}'_0 = \mathbf{X}\sin\Theta + \mathbf{G}\cos\Theta \quad (127)$$

are applied to equation (125), it goes over into

$$[-i\gamma^\mu\partial_\mu + m]\psi' + [-Q\gamma^k A_k - Q'\gamma^k Z_k + Q(\gamma^k\gamma^5)G_k + Q'(\gamma^k\gamma^5)X_k]\psi' = 0 \quad (128)$$

with

$$Q := \frac{1}{2}(\sigma^3 + \sigma^0)e, \ e = (gg')^{1/2}(|g| + |g'|)^{1/2},$$
$$Q' := -\frac{1}{2}\left[\left(\frac{g}{g'}\right)^{1/2}\sigma^3 - \left(\frac{g'}{g}\right)^{1/2}\sigma^0\right]e. \quad (129)$$

Owing to (116) the Weinberg transformation applies to the E' and B' fields too. But for the sake of brevity we do not give the explicit formulas.

Equation (128) admits the derivation of a conserved current $\bar\psi\gamma^\mu\psi$ which simultaneously leads to the conservation of leptonic probability and of electric charge. Owing to the mass m in (128) no conserved magnetic current exists. This fact allows for the creation and annihilation of magnetic charges and is in accordance with the hypothesis of Section 1. In addition this fact is compatible with the effective dynamics of the vector fields: The latter is defined by the canonical equations (112)–(115). To complete the theory of vector fields their constraints have to be formulated (electronic and magnetic Gauß law). In the canonical version of the theory these constraints need not be postulated, but can be derived from equations (112)–(115) in combination with the spinor equation (128) (compare for instance [9], section 8.2). This procedure guarantees that no contradiction can arise between current conservation induced by the vector field dynamics and current conservation induced by the spinor field dynamics.

In summary it may be said: From (128) it follows that the negatively charged electrons aquire an additional positive magnetic charge under the influence of the CP-symmetry breaking, while neutrinos do not take part in this transmutation of electrons. Of course, also positrons are subjected to this transmutation while antineutrinos remain unchanged. The structure of the effective theory describes an extension of the electroweak Standard model to comprise leptonic dyons, but at the expense of an immediate $SU(2)$-symmetry breaking. The same effect of CP-symmetry breaking can be expected for quarks and will be treated elsewhere. A conserved current exists only for the electric charge and probability density of the fermions. The fact that the existence of dyons is connected with CP-symmetry breaking was also deduced in phenomenology [29].

The above derived results could possibly support the assumption of Rukhadze et al. [30], that electrons play an essential role in the set-off of nuclear reactions since in case of CP-symmetry breaking which corresponds to the experimental arrangement the additional magnetic charge of the leptons may exert a considerable influence on the rate of such reactions. The absence of a conserved magnetic current explains at the same time why magnetic monopoles have not been observed so far.

Concerning Lochak's hypothesis of magnetically charged neutrinos [31], it is not excluded that such particles can be discovered in the above formalism, too, if some of the assumed symmetry constraints of the present calculation are lifted in more detailed calculations to be done elsewhere.

Acknowledgement

I would like to thank Prof. Peter Kramer for a critical reading and discussion of the manuscript.

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